ANALYSIS OF THE THREE-PHASE, CAGE ROTOR, INDUCTION MOTOR OPERATING WITH UNBALANCED APPLIED VOLTAGES

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A THESIS

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Sam Price Gullatt, Jr.

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CHAPTER I

SYMMETRICAL COMPONENTS--GENERAL THEORY

In general, analytical solutions of polyphase electrical circuits are quite readily obtained through the application of Kirchhoff's laws and the principle of superposition. While these methods are applicable to linear, balanced polyphase systems, difficulties soon develop in many of the unbalanced or unsymmetrical networks; and solutions of these networks by the above methods alone become quite involved and often tedious.

Too, the principle of superposition requires that the components of the system remain unaltered under all conditions of a specific problem. It is unfortunate, from the analytical point of view, that this linear condition does not prevail in circuits where unbalanced rotating electrical machines are a constituent. The polyphase induction motor operating with unbalanced applied voltages is an example of such a system.

Analytical solutions of the operating characteristics of the polyphase induction motor with balanced applied voltages are normally obtained through the direct application of Kirchhoff's laws and the superposition principle. If by some means, therefore, when unbalanced terminal voltages are applied to the motor the unbalanced terminal voltages can be resolved into a balanced system, or systems, of potentials, the problem is reduced to one of quite familiar nature. Thus, the method of symmetrical components, due to C. L. Fortescue,[1] is a powerful tool when used to obtain analytical solutions for the operation of the polyphase induction motor with unbalanced applied voltages, for in this method the unbalanced system of applied voltages is resolved into several balanced systems.

Fortescue's theorem (applied to a three-phase system of vectors) states that any unbalanced, three-phase system of vectors may be resolved into these balanced systems of vectors, namely:

(a) Positive-sequence system. A balanced, three-phase system of vectors which has the same phase sequence as the original unbalanced system of vectors.

(b) Negative-sequence system. A balanced, three-phase system of vectors which has a phase sequence opposite to that of the original unbalanced system of vectors.

(c) Zero-sequence system. A system of three, single-phase vectors all of equal magnitude and having the same time-phase position with respect to any given reference axis.

To illustrate this theorem, consider the unbalanced system of vectors of Figure 1. The system of vectors may be resolved into the three balanced systems of Figure 2 as is verified by Figure 3. The positive direction of rotation is taken as counterclockwise.

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[1] Fortescue, C. L., "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks," Transactions A. I. E. E., Vol. 37, Pt. 2, p 1027.

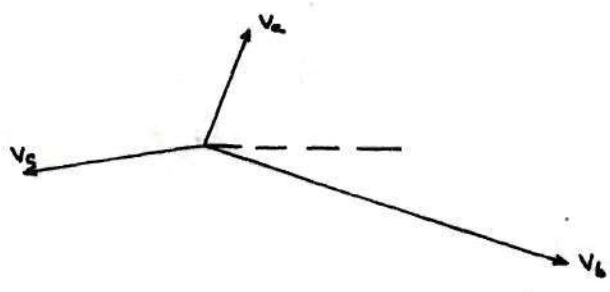
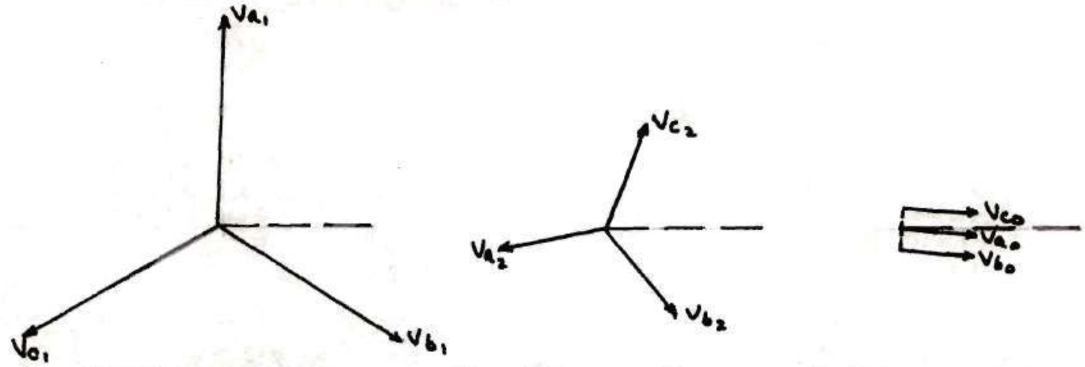


Figure 1. Unbalanced system of vectors.



Positive- sequence system

Negative- sequence system

Zero- sequence system

Figure 2. Sequence systems of vectors comprising the original unbalanced system.

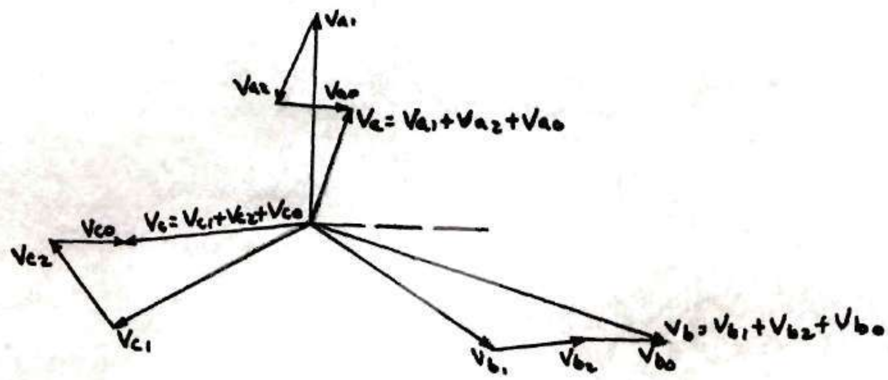


Figure 3. Illustration of the manner in which the sequence components combine to form the original unbalanced system.

In order that the sequence systems of Figure 2 be satisfied, the following vector relationships must exist.

For the Positive-Sequence System:

$$V_{a1} = V_{a1}$$
$$V_{b1} = V_{a1}\,\epsilon^{j240^\circ}$$
$$V_{c1} = V_{a1}\,\epsilon^{j120^\circ}$$

For the Negative-Sequence System:

$$V_{a2} = V_{a2}$$
$$V_{b2} = V_{a2}\,\epsilon^{j120^\circ}$$
$$V_{c2} = V_{a2}\,\epsilon^{j240^\circ}$$

For the Zero-Sequence System:

$$V_{a0} = V_{b0} = V_{c0}$$

To avoid the cumbersome exponential form of writing vector relationships, Fortescue introduced an operator, a, which, when operating on a vector, rotated that vector through 120 degrees in the positive direction. In a similar manner, the operator a^2, when operating on a vector, rotated that vector through 240 degrees in the positive direction. It is interesting to note two other properties of these operators, namely

$$(1 + a + a^2)V = 0$$

and $$(aV)(a^2V) = a^3V = V$$

Making use of these operators, the various vector relationships for the positive-and negative-sequence systems may be rewritten:

For Positive-Sequence System

$$V_{a1} = V_{a1}$$
$$V_{b1} = V_{a1}\epsilon^{j240^\circ} = a^2 V_{a1}$$
$$V_{c1} = V_{a1}\epsilon^{j120^\circ} = a V_{a1}$$

For Negative-Sequence System

$$V_{a2} = V_{a2}$$
$$V_{b2} = V_{a2}\epsilon^{j120^\circ} = a V_{a2}$$
$$V_{c2} = V_{a2}\epsilon^{j240^\circ} = a^2 V_{a2}$$

Referring once again to Figure 3 it is seen that the following relations are valid:

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{1}$$
$$V_b = V_{b1} + V_{b2} + V_{b0} \tag{2}$$
$$V_c = V_{c1} + V_{c2} + V_{c0} \tag{3}$$

Rewriting, making use of vector relationships given above:

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{4}$$
$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \tag{5}$$
$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \tag{6}$$

Solving this system of equations for V_{a1}, V_{a2}, and V_{a0} gives:

$$V_{a1} = 1/3(V_a + a V_b + a^2 V_c) \tag{7}$$
$$V_{a2} = 1/3(V_a + a^2 V_b + a V_c) \tag{8}$$
$$V_{a0} = 1/3(V_a + V_b + V_c) \tag{9}$$

It is possible, therefore, to resolve a system of unbalanced three-phase vectors into three balanced systems of vectors. Solutions for V_{a1}, V_{a2}, and V_{a0} may be obtained by solution of the above equations or by graphical means.[3] Both current and voltage vector systems may be solved in this manner.

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[3] Solution of positive-and negative-sequence components by graphical means appears in Appendix I in conjunction with the solution of an actual problem.

The question now arises as to what meaning shall be attached to positive-, negative-, and zero-sequence impedance components. Positive-sequence impedance shall be construed to mean that impedance offered to a balanced positive-sequence system of voltages. Negative-and zero-sequence impedance refers to the impedance offered to systems of negative- and zero-sequence voltages, respectively.

It is important to remember that there is no interaction between the various sequence systems.

CHAPTER II

OPERATION OF THE THREE-PHASE INDUCTION MOTOR WITH BALANCED APPLIED VOLTAGES

Before discussing the operation of the three-phase induction motor with unbalanced applied voltages, a brief discussion of the operating characteristics of the machine with balanced applied voltages should be made.

With a balanced system of voltages applied to the stator terminals of a symmetrically wound motor, a circular magnetic field is set up in the air gap. This follows from the fact that, since the stator winding is symmetrical, the individual phases are separated by 120 electrical degrees and the flux produced by the current flowing in the individual windings is varying sinusoidally with respect to time. The vector addition of the flux produced by the three individual phases gives a resultant vector of fixed magnitude which rotates at uniform angular velocity. The direction of rotation of the resultant vector depends upon the phase sequence of the applied potentials in addition to the particular winding pattern for any particular machine.

Since the direction of rotation of the air gap flux in any particular machine is dependent upon the phase sequence of the applied potentials, it is quite apparent that the direction of rotation of this flux may be changed simply by changing the phase sequence of the applied voltages. Since the rotor, if free to turn, rotates in the same direction as the rotating magnetic field in the air gap, the direction of rotation of the rotor may also be changed by reversal of the phase sequence of the applied voltages.

The velocity of rotation of the air gap flux is directly proportional to the frequency of the impressed voltages, and inversely proportional to the number of poles for which a particular machine is wound. The speed at which the air gap field rotates in revolutions per unit time is called the synchronous speed of the machine.

It is a well-known fact that changes in shaft load result in changes in speed of rotation of the rotor. Since the angular velocity of the air gap flux is constant (frequency of the impressed potentials assumed constant), it is quite evident that there exists a speed differential between the rotor and the revolving air gap field. This differential in speed is the revolutions slip (per unit time) of the machine for any particular rotor speed. It is conventional to express the revolutions of slip as a fraction of the synchronous speed, slip, $S = \dfrac{N_s - N}{N_s}$ (10)

With the rotor at standstill (blocked), the slip, from the definition, is unity; therefore, the frequency of the currents which flow in the rotor circuit will be the same as the frequency of the stator currents (or the stator magnetic field). If, on the other hand, the rotor turns at synchronous speed, there is no slip.

As a consequence of the foregoing discussion, it is reasonable to state that, for rotor speeds between standstill and synchronous speed, the frequency of the rotor currents is equal to the stator frequency multiplied by the slip of the motor.

With the rotor blocked, the voltage induced in a rotor inductor, due to the air gap flux, is:

$$e = Blv_s \text{ abvolts/inductor} \tag{11}$$

or $E_{m20} = B_m l v_s \times 10^{-8}$ volts/inductor

$$E_{20} = 0.707\, B_m l v_s \times 10^{-8} \text{ volts/inductor} \tag{12}$$

With the rotor free to rotate, however, the velocity of the rotor inductor, with respect to the rotating air gap flux, is $V_r = V_s - V$, which is the slip of the motor in centimeters per second. Thus, the voltage induced in the rotor inductor becomes:

$$E_2 = 0.707\ B_m\ l\ (V_s - V) \times 10^{-8} \text{ volts/inductor.}$$

Multiplying both top and bottom of this equation by the velocity of the rotating field,

$$E_2 = 0.707\ B_m\ l\ V_s\ \frac{(V_s - V_n)}{V_s} \times 10^{-8} \text{ volts/inductor.}$$

However, $\quad \text{Slip, } S = \dfrac{V_s - V}{V_s}$

Therefore, for operation at any slip S, the emf induced in the rotor of the loaded motor is given by:

$$E_2 = SE_{20} \tag{13}$$

where E_{20} is the emf, in volts per phase, induced in the rotor bars at standstill. The equation holds when the slip of the motor is not excessive.

The power input to the motor on a per-phase basis is given by:

$$P_2 = I_1 V_1 \cos \theta_1 \quad \text{watts/phase} \tag{14}$$

Since the power loss in the stator is the stator-copper loss per phase plus one-third the total iron loss of the machine, the power across the air gap may be given

$$P_2 = P_1 - I_1^2 R_1 - 1/3 \text{ total iron loss} \quad \text{watts/phase,} \tag{15}$$

or the power input to the motor may be written

$$P_2 = I_2 E_2 \cos\theta_2 \quad \text{watts/phase} \tag{16}$$

At standstill, the rotor current, I_2, per phase is given by:

$$I_2 = \frac{E_{20}}{\sqrt{(R_2)^2 + (2\pi f_2 L_2)^2}} \quad \text{amperes} \tag{17}$$

and the rotor power factor

$$\cos\theta_2 = \frac{R_2}{\sqrt{(R_2)^2 + (2\pi f_2 L_2)^2}} \tag{18}$$

It must be understood that the rotor components are referred into the stator as in the case for the static transformer. It is possible to make this transformation for the polyphase induction motor is, in theory, equivalent to a static transformer operating with a non-inductive load. The turns ratio for the induction motor is

$$n = \frac{m_1 N_1 k_{p1} k_{d1}}{m_2 N_2 k_{p2} k_{d2}}$$

m = number of phases for which the machine is wound
N = actual turns per phase
k_p = pitch factor
k_d = distribution factor

where the subscripts 1 and 2 refer to the stator and rotor, respectively.

With the motor operating at any slip, S, the rotor current per phase is

$$I_2 = \frac{SE_{20}}{\sqrt{(R_2)^2 + (2\pi f L_2)^2}} \quad \text{amperes}$$

but from previous discussion $f_2 = sf_1$

$$I_2 = \frac{SE_{20}}{\sqrt{(R_2)^2 + (2\pi Sf_1L_2)}} \quad \text{amperes} \tag{20}$$

$$\text{or} \quad I_2 = \frac{SE_{20}}{\sqrt{(R_2)^2+(S\ X_2)^2}} \quad \text{amperes} \tag{21}$$

where $X_2$is the reactance of the rotor at stator frequency.

Rewriting equation (21)

$$I_2 = \frac{E_{20}}{\sqrt{\left(\frac{R_2}{S}\right)^2 + X_2^{\ 2}}} \quad \text{amperes} \tag{22}$$

This equation indicates that the motor operating at a slip, S, has an apparent rotor resistance $R_2/\ S$ ohms per phase.

Making use of the deduction above (i. e., the apparent rotor resistance is the actual rotor resistance divided by the slip of the motor), the rotor input is seen to be

$$P_2 = I_2^{\ 2} \frac{R_2}{S} \quad \text{watts/phase} \tag{23}$$

Since the actual rotor power loss is $I_2\ R_2$ watts per phase, the power developed (i. e., pulley power plus friction and windage loss) is given by

$$P_d = I_2^{\ 2} \frac{R_2}{S} - I_2^{\ 2} R_2 \quad \text{watts/phase} \tag{24}$$

$$\text{or} \quad P_d = I_2^{\ 2} \frac{R_2}{S} (1 - S) \quad \text{watts/phase}$$

from which it is seen that

$$P_d = P_2\ (1 - S) \quad \text{watts phase} \tag{25}$$

and the power at the pulley is

$$P_p = P_2\ (1 - S) - 1/3 \text{ Total friction and windage loss} \tag{26}$$

Equation (26) is an expression for the pulley power in watts per phase. Making use of the superposition principle, it is seen that the pulley power is three times the pulley power per phase. The pulley torque and horsepower output are easily obtained from the total pulley power in watts.

$$\text{Horsepower} = \frac{\text{Watts}}{746} \tag{27}$$

$$\text{Pulley Torque} = \frac{\text{Horsepower x 33000}}{2 \ \text{ x Rotor speed in revolutions per minute}} \quad \text{pounds-feet} \tag{28}$$

Previously, it was mentioned that the induction motor is equivalent to the static transformer operating with a non-induction load. It is quite logical to assume, therefore, that the transformer vector diagram and equivalent circuit are applicable with certain modifications, for the polyphase induction motor.

From equation (22) above, it was seen that the apparent rotor resistance in ohms per phase was the actual rotor resistance in ohms per phase divided by the slip of the motor. This is the apparent resistance at any particular slip, and the mechanical power developed per phase within the machine is given by equation (23) as

$$P_d = I_2^2 \frac{R_2}{S} - I_2^2 R_2 \text{ watts/phase}$$

$$\text{or } P_d = I_2^2 R_2 \frac{(1 - S)}{S} \tag{29}$$

The mechanical load on the machine may, therefore, be represented in the equivalent circuit as a pure resistance, the value of which is $R_2 \frac{(1 - S)}{S}$ ohms per phase.

The component parts of the equivalent circuit may be determined from no-load and blocked rotor laboratory tests of any particular motor.[1]

The equivalent circuit for the induction motor appearing below is the same as for a static transformer with one exception; namely, the method of representing a load on the machine.

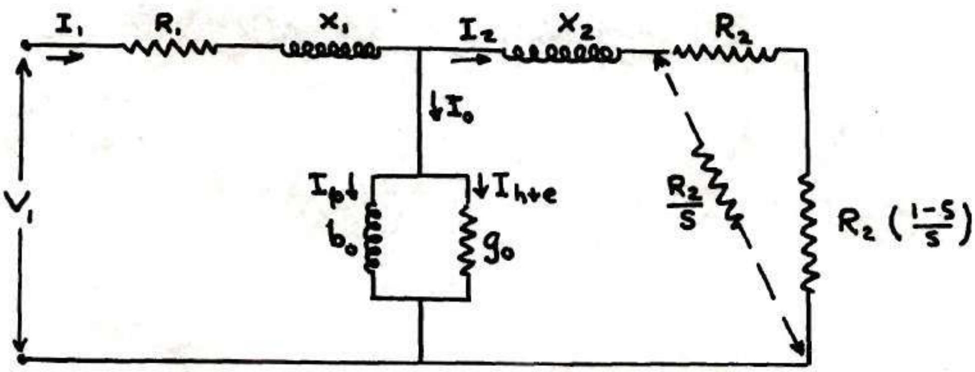

FIGURE 4. Exact Equivalent Circuit for Polyphase Induction Motor. All Components Given on a Per-Phase Basis.

The resistances in the rotor branch may be added together to give the apparent rotor resistance, $\frac{R_2}{S}$ ohms per phase.

1 Determination of the machine constants for a particular machine is given in Appendix II.

CHAPTER III

OPERATION OF THE THREE-PHASE INDUCTION MOTOR WITH UNBALANCED APPLIED VOLTAGES

In the preceding chapter, it was seen that with a balanced system of potentials applied to the stator terminals, the resulting stator currents set up a revolving magnetic field in the air gap. This field revolves at synchronous speed; the direction of rotation is dependent upon the phase sequence of the applied potentials. The currents induced in the rotor circuit by this air gap flux produces a torque in the same direction as the rotating field.

Previously (Chapter I), it was seen that an unbalanced system of voltage vectors may be resolved into balanced positive-, negative-, and zero-sequence systems of vectors. Only positive-and negative-sequence systems appear if the unbalanced vector system forms a closed triangle (i. e., there is no neutral or ground return), for the zero-sequence system disappears as a consequence of equation (9). Thus, with a three-wire, three-phase system, only positive-and negative-sequence vector systems need be considered.

Therefore, from the foregoing, with a system of unbalanced voltages applied to the stator terminals, rotating magnetic fields will be established by both the positive-and negative-sequence voltage systems. The resulting air gap field is no longer circular in nature, but has become elliptical. The performance of the machine may be calculated, however, by computing the effect of each system separately and superimposing the results.[1]

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[1] Slepian, J., "Induction Motors on Unbalanced Voltages," Electrical World, p 313, 1920.

Since the air gap fields, due to the two sequence systems, rotate in opposite directions, the torque produced by each will be in opposite directions.

An equivalent circuit is established for the positive-sequence system and an equivalent circuit is also established for the negative-sequence systems. Since the two systems are balanced and since there is no interaction between the positive-and negative-sequence systems, the total performance is determined by superimposing the results calculated from the two equivalent circuits.

If the rotor turns at synchronous speed in one direction (assume positive direction), the slip of the positive-sequence system (from Chapter II) is zero; while the slip of the negative-sequence system is 2 (i. e., the negative-sequence air gap flux is rotating at synchronous speed. Relative motion between the two is, therefore, twice synchronous speed). With the rotor operating at slip, S, the negative-sequence slip is quite obviously 2 - S.

The frequency of the negative-sequence rotor currents is apparently

$$f_2 = (2 - S)f_1 \tag{30}$$

or, at small values of slip, S, the rotor currents are approximately twice the frequency of the stator currents. The actual rotor resistance and reactance must be modified due to skin effect produced by the double frequency currents. "The change in rotor reactance is small and usually neglected";[2] however, this is not the case for the actual rotor resistance. The actual rotor resistance (for the negative-sequence system) varies

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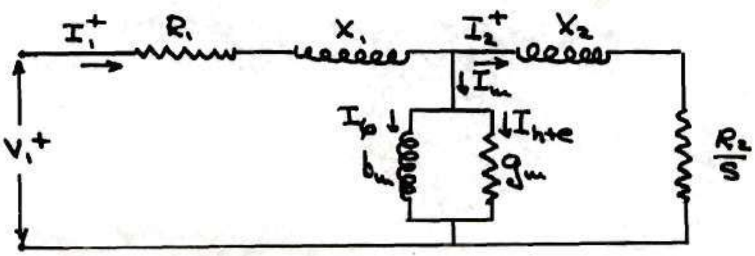
[2] Dahl, O. G. C., Electric Circuits, Vol. I, (McGraw-Hill Book Company, 1928), p 143.

considerably for different machines, depending upon size and shape of the rotor bars. For small machines, the actual rotor resistance is predicted by multiplying the ohms resistance by the factor, 1.80.[3] This is a mean value for several small machines tested.

The stator resistance is the same for both sequence systems and may be determined by the method given in Appendix II. The leakage reactance of stator is also taken as the same for both sequence systems.

Since it is very difficult to determine to any degree of accuracy the exact magnetizing branch of both the positive-and negative-sequence systems, the same magnetizing branch components are normally taken as the same for both sequence systems. Some authors[4] suggest that the negative-sequence system may be neglected entirely without introducing appreciable errors in the calculated results.

The equivalent circuit diagrams may now be constructed as in Figure 5.



(a) Positive-Sequence System

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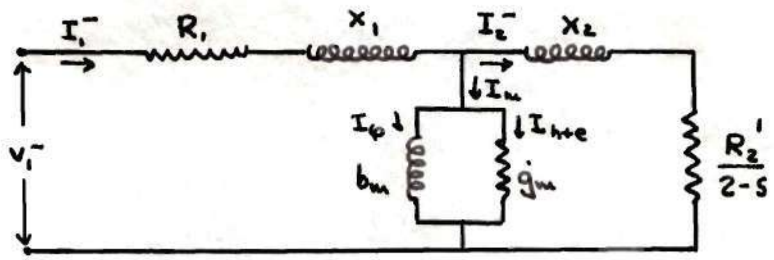
[3] Dahl, O. G. C., Electric Circuits, Vol. I, (McGraw-Hill Book Company, 1928), pp 138-144. Lawrence, R. R., Principles of Alternating Current Machinery, (McGraw-Hill Book Company, 1940), pp 579-582. Lyon, W. V., Applications of the Method of Symmetrical Components, (McGraw-Hill Book Company, 1937), pp 295-308.

[4] Loc. cit.



(b) Negative-Sequence System

FIGURE 5. The equivalent circuit diagrams for the (a) positive-sequence and (b) negative-sequence systems for symmetrically constructed three-phase induction motor operating with unbalanced applied potentials. Components are on per phase basis.

The potentials applied to the positive-and negative-sequence equivalent circuits may be obtained from the original unbalanced system by graphical means as outlined in Appendix I. The two equivalent circuits are solved individually to determine the torque resulting from each system and the results then superimposed.

The stator currents at any slip, S, is by

$$I_1^+ = \frac{V_1^+}{Z^+} \tag{31}$$

$$I_1^- = \frac{V_1^-}{Z^-} \tag{32}$$

where Z^+ is the equivalent impedance of the positive-sequence system, and Z^- is the equivalent impedance of the negative-sequence system.

$$I_2^+ = I_1^+ \frac{(Z_m)}{(Z_m + Z_2^+)} \tag{33}$$

$$I_2^- = I_1^- \frac{(Z_m)}{(Zm + Z_2^-)} \tag{34}$$

where Z_m is the impedance of the magnetizing branch.

The developed torque of the two sequence systems is given by

$$T_2^{+} = \frac{p}{4\pi f_1}(I_2^{+})^2 \frac{R_2}{S_1}\frac{(550)}{(746)} \text{ pounds-feet/phase} \qquad (35)$$

$$T_2^{-} = \frac{p}{4\pi f_1}(I_2^{-})^2 \frac{R_2'}{2 - S}\frac{(550)}{(746)} \text{ pounds-feet/phase} \qquad (36)$$

By application of the superposition theorem, the total net torque developed is

$$T = 3\left[T^{+} - T^{-}\right] \text{ pounds-feet/phase} \qquad (37)$$

Pulley power is given by

$$P_p = 2\pi \text{x Speed in RPM x } T - P_{f+w}$$

$$P_p = 2\pi \text{ x } \frac{2f}{p}(1 - S)\; T - P_{f+w} \text{ foot pounds per second} \qquad (38)$$

and $\text{Horsepower Output} = \dfrac{\text{foot pounds per second}}{550}$ (39)

The above equations (31) through (39) follow from the work presented in Chapter II.

The total copper loss is the summation of the copper losses of the positive-and negative-sequence systems.

Total positive-sequence copper loss

$$C_L^{+} = 3\left[(I_1^{+})^2 R_1 + (I_2^{+})^2 R_2\right] \text{watts} \qquad (40)$$

Total negative-sequence copper loss

$$C_L^{-} = 3\left[(I_1^{-})^2 R_1 + (I_2^{-}) \; R_2'\right] \text{watts} \qquad (41)$$

The friction and windage losses are determined from the no-load conditions as in Appendix II.

The core loss for the motor operating with unbalanced applied potentials presents a very difficult problem due to the fact that the resultant air-gap field is elliptical in nature. Approximations to the total core loss is given by Lyon and his method of predicting this loss will be given here.[5]

Since the core loss is proportional to the square of the voltage, the total stator core loss is given by

$$3\left[\left(I_2^+ \; Z_2^+\right)^2 g_0 + \left(I_2^- \; Z_2^-\right)^2 g_0\right] \text{ watts} \tag{42}$$

where g_0 is the conductance of the magnetizing branch. The core loss in the rotor is estimated to be 0.045 times the stator core loss. The total core loss for the machine may be obtained, approximately, by adding the stator and rotor core losses.

5 Lyon, W. V., op. cit., pp 283-286 and pp 306-308.

CHAPTER IV

SINGLE-PHASE OPERATION OF THE THREE-PHASE INDUCTION MOTOR

A condition bringing about bad unbalance of the applied voltages to an induction motor is the case where one of the three input lines to the machine is opened. A single-phase voltage is thereby applied at the two remaining stator terminals.

Single-phase operation of a three-phase induction motor is a special case of the condition of operation with unbalanced applied voltages. The impedance diagrams for the positive-and negative sequence systems are, therefore, the same as those appearing in Chapter III.

A problem arises, however, in determining the voltage impressed across the two sequence systems. Quite obviously the current flowing in the opened phase is zero; so, with no neutral or ground return, the zero-sequence currents are, by definition, zero

$$\begin{aligned} I_{a0} &= 1/3\ (I_A + I_B + I_C) \\ 0 &= 1/3\ (0 + I_B + I_C) \\ \text{or} \quad I_C &= -\ I_B \end{aligned} \tag{43}$$

This, of course, assumes phase A to be opened.

From equation (7) of Chapter I

$$I_{1a}^{+} = 1/3\ (\bar{I}_A + a\,\bar{I}_B + a^2\,\bar{I}_C) \tag{7}$$

$$I_{1a}^{+} = 1/3\ (0 + a(I + j0) + a^2(-I + j0))$$

$$I_{1a}^{+} = \frac{1}{\sqrt{3}}\ (jI) \text{ amperes} \tag{44}$$

where I is the line current.

$$\text{and} \quad I_{1a}^{-} = 1/3\ (\bar{I}_A + a^2 \bar{I}_B + a \bar{I}_C) \tag{8}$$

$$I_{1a}^{-} = 1/3\ (0 + a^2 (I + j_0) + a(-I + j_0))$$

$$I_{1a}^{-} = \frac{1}{\sqrt{3}} (-jI) \ \text{amperes} \tag{45}$$

(Subscript 1 refers to stator currents.)

From equations (44) and (45), the stator positive-sequence and negative-sequence currents are equal in magnitude. The stator current for either sequence system, however, is made up of the exciting currents and the rotor current added vectorially. From Chapter III, it was stated that for small values of slip, the negative-sequence exciting currents may be neglected without introducing appreciable error into the calculations.

In the open-circuited phase, it is apparent that since there is no resultant current flow, the vector summation of the positive-and negative-sequence currents in this phase must be zero. We may now write an expression for the voltage across the open-circuited phase, keeping in mind the definition for sequence impedance given in Chapter I.

$$V_{1a}^{+} = I_{1a}^{+} (R_1 + jX_1) + I_{2a}^{+} \left(\frac{R_2}{S} + jX_2\right) \tag{46}$$

$$V_{1a}^{-} = I_{1a}^{-} (R_1 + jX_1) + I_{2a}^{-} \left\{\frac{R_2'}{2 - S} + jX_2\right\} \tag{47}$$

The only known voltage, however, is the voltage between terminal B and C. The voltages across the active phases (i. e., V_{1B} and V_{1C}), expressed in terms of their positive-and negative-sequence components, are written

$$V_{1B} = V_{1B}^{+} + V_{1B}^{-} \tag{48}$$

$$V_{1C} = V_{1C}^{+} + V_{1C}^{-} \tag{49}$$

$$\text{also} \quad V_{BC} = V_{BC}^{+} + V_{BC}^{-} \tag{50}$$

or, rewriting the expression for V_{BC} in terms of V_{1A} and V_{1A} ,

$$V_{BC} = j\sqrt{3}\,(V_{1A}^{+} - V_{1A}^{-}) \qquad (51)$$

since, $V_{BC}^{+} = j\sqrt{3}\,V_{1A}^{+}$ and $V_{BC}^{-} = -j\sqrt{3}\,V_{1A}^{-}$

Rewriting,

$$V_{BC} = j\sqrt{3}\left\{\left[I_{1a}^{+}\,(R_1 + jX_1) + I_2^{+}\left(\frac{R_2}{S} + jX_2\right)\right] + \left[I_1^{-}\,(R_1 + jX_1) + I_2^{-}\left(\frac{R_2{}'}{2 - S} + jX_2\right)\right]\right\} \qquad (52)$$

From previous discussion,

$$I_1^{+} = I_2^{+} + I_0^{+} \qquad (53)$$

$$I_1^{-} = -I_1^{+} = -(I_2^{+} + I_0^{+}) \qquad (54)$$

then, $$I_2^{+} = I_1^{+} - I_0^{+} \qquad (55)$$

and, $$I_2^{-} = I_1^{-} = -I_1^{+} \qquad (56)$$

Here, the magnetizing component of the negative-sequence system is neglected.

Rewriting equation (52),

$$V_{BC} = j\sqrt{3}\left[I_0^{+}\,(R_1 + jX_1) + I_2^{+}\,(R_1 + jX_1) + I_2^{+}\left(\frac{R_2}{S} + jX_2\right) + I_n^{+}\,(R_1 + jX_1) + I_2^{+}\,(R_1 + jX_1) + I_0^{+}\left(\frac{R_2{}'}{2 - S} + jX_2\right) + I_2^{+}\left(\frac{R_2{}'}{2 - S} + jX_2\right)\right] \qquad (57)$$

$$V_{BC} = j\sqrt{3}\left[2I_0^{+}\,(R_1 + jX_1) + I_0^{+}\left(\frac{R_2{}'}{2 - S} + jX_2\right) + 2I_2^{+}\,(R_1 + jX_1) + I_2^{+}\left(\frac{R_2}{S} + \frac{R_2{}'}{2 - S} + j2X_2\right)\right] \qquad (58)$$

$$V_{BC} = j\sqrt{3}\Big[I_0^+ \Big(2R_1 + \frac{R_2'}{2-S}\Big) + jI_0^+ (2X_1 + X_2) + I_2^+ \Big(2R_1 + \frac{R_2}{S} + \frac{R_2'}{2-S}\Big) + j2I_2^+ (X_1 + X_2) \Big]$$

Since I_0 Z_0 is very nearly 180 degrees out of phase with V_{BC}, it may be subtracted arithmetically from V_{BC}.[1] The expression for the positive-sequence rotor current may be written as

$$I_2^+ = \frac{\dfrac{-jV_{BC}}{\sqrt{3}} - I_0^+ \Big[\Big(2R_1 + \dfrac{R_2'}{2-S}\Big) + j(2X_1 + X_2)\Big]}{2R_1 + \dfrac{R_2}{S} + \dfrac{R_2'}{2-S} + j2(X_1 + X_2)} \text{ amperes} \qquad (59)$$

The total internal torque developed,

$$T = T^+ - T^- = 3\,\frac{P}{4\pi f_1}\Big[(I_2^+)^2 \frac{R_2}{S} - (I_2^-)^2 \frac{R_2'}{2-S} \Big] \frac{550}{746} \text{ pounds-feet} \qquad (60)$$

Pulley power is, therefore,

$$P_p = 2\pi \text{ (speed in RPS) } T - \text{Friction and windage loss}$$

$$P_p = 2\pi \frac{2f}{p}(1 - S)\, T - P_{f+w} \qquad (61)$$

Total copper loss is given by:

$$C_L = 3\Big[(I_1^+)^2 R_1 + (I_1^-)^2 R_1 + (I_2^+)^2 R_2 + (I_2^-)^2 R_2' \Big] \text{ watts} \qquad (62)$$

From these equations, the performance of the three-phase induction motor may be calculated when the motor is operating single-phase, (i. e., one terminal is open).

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[1] Lawrance, R. R., Principles of Alternating-Current Machinery, (McGraw-Hill Book Company, 1940), p 572.

CHAPTER V

DISCUSSION OF LABORATORY TEST RESULTS AND CALCULATED PERFORMANCE OF A 5-HORSEPOWER, CAGE ROTOR, THREE-PHASE INDUCTION MOTOR

Load tests were made on a 5-horsepower, cage rotor, three-phase induction motor with balanced applied potentials and for a wide range of unbalanced applied potentials. The machine was also operated single-phase (by disconnecting one of the supply terminals); for this is the limiting case of unbalanced applied potentials. The data obtained from these load runs was then corrected by application of the meter calibration curves and appears in Tables I through X, inclusive.

The machine was loaded by a prong brake constructed especially for these tests by the author.[1] This prong brake, damped by a mechanical vibration filter system (dash pot-spring combination) held the load on the machine quite constant. The author is convinced that the tests were made under the very best conditions possible and that the accuracy involved is on a high level.

Determination of the machine constants presented a very definite problem.[2] Since the rotor was of single cage construction, it was not possible to measure directly the rotor resistance. As a consequence, the rotor resistance was calculated from design specifications as shown in Appendix II.

Another problem - the determination of the stator and rotor leakage reactances - was solved by plotting the blocked rotor test as

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[1] See Acknowledgements.

[2] See Appendix II.

in Figure L. Since the "short-circuit current versus terminal voltage" curve is a straight line, it was assumed that, without introducing appreciable error, the leakage reactance of the stator was equal to the leakage reactance of the rotor. This assumption may be a source of error; however, the author believes the error introduced is quite small.

Determination of the core loss with the machine operating with unbalanced applied potentials is, as pointed out in Chapter III, quite difficult. Numerous references were consulted on this problem; however, the prediction suggested by Lyon seemed quite plausible and is, therefore, the method used in this thesis.[3] The problem of exact determination of the core loss, due to an elliptic air gap field, is quite involved and the author suggests this problem as a very excellent thesis project.

From the curves and tabulated data, it is seen that predictions of the operating characteristics for an induction motor (operating with unbalanced applied potentials) may be made through the use of the methods outlined in the test with a high degree of accuracy. The discrepancy between accurately measured characteristics and those obtained through calculation is on the order of 3 to 5 per cent. It is suggested that the magnetizing branch be included in the negative-sequence equivalent circuit; for its negligence in several "trial" calculations resulted in discrepancies of 1 to 3 per cent in the equivalent negative-sequence impedance calculated from circuits containing this branch.

In the interest of conserving time, it is suggested that the impedance for the positive-and negative-sequence systems be computed for a wide range of operating speeds. These impedances may then be plotted

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3
 See page 19 of this Thesis.

on a curve (impedance versus slip) and if the resulting curve is smooth, it is unlikely that the error may be introduced due to error in calculation of impedance. This scheme was followed in this thesis and it was found that calculations could be made quite speedily for the various degrees of unbalance (i. e., different systems of unbalanced applied potentials).

An interesting result of this is that, although the negative-sequence torque is quite small, the presence of the negative-sequence currents makes it necessary to operate the machine at greatly reduced output to avoid overheating. As a consequence of this, care should be exercised to insure applied potentials being as nearly balanced as possible.

A P P E N D I X E S

APPENDIX I

GRAPHICAL METHOD OF DETERMINATION OF POSITIVE-AND NEGATIVE-SEQUENCE VECTOR SYSTEMS

In Chapter I, formulae were developed whereby a system of unbalanced vectors could be resolved into three balanced systems of vectors. It will be recalled, however, that it was necessary to know the vector relationships of the unbalanced system. Three relationships may be obtained from graphical construction, provided the system of vectors form a closed triangle. In Chapter III, mention was made of the fact that there is no zero-sequence system present in the particular case where the unbalanced vector system forms a closed triangle. It is necessary, therefore, to confine our attention to the determination of the positive-and negative-sequence components, since in all cases considered in this work, the unbalanced vector systems form a closed triangle (i. e., there are no neutral or ground returns present).[1]

Since it is necessary to construct a vector diagram of the applied voltage system in order to obtain the correct time-phase relationships of the component vectors, the positive-and negative-sequence systems may be determined graphically from the vector diagram of the unbalanced vector system. Since considerable time is saved and the accuracy is quite good when appropriately large scale drawings are made (i. e., one volt per millimeter), graphical means of determining the

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[1] Methods for determining the zero-sequence vector system by graphical means are given quite completely in Symmetrical Components by Wagner, C. F., and Evans, R. D., (McGraw-Hill Book Company, 1933), pp 259 - 264.

various sequence components are used in the analytical solutions of this thesis. The method is given below.[2]

The terminal voltages, V_a, V_b, and V_c are read from voltmeters at the machine terminals. One voltage, such as V_a, is selected as the reference voltage and is laid down on the horizontal with its proper magnitude (to scale). Arcs are swung from either end of this vector with the magnitudes of V_b and V_c as radii. Figure 6 shows this construction.

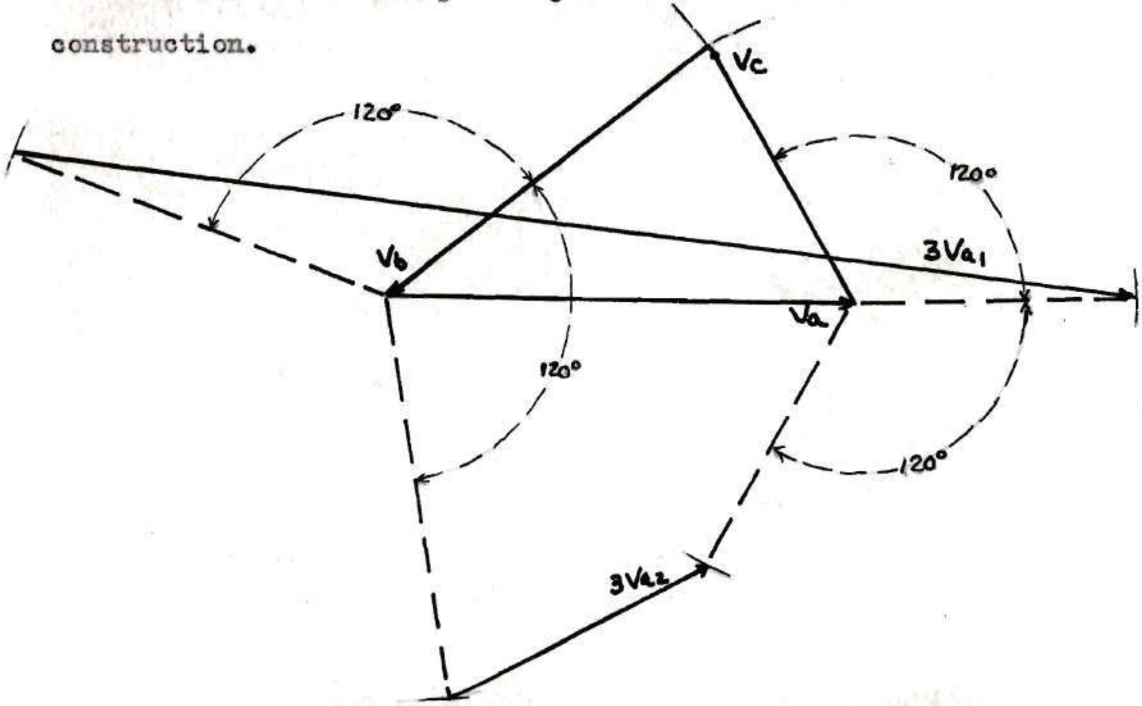

FIGURE 6. Graphical Determination of Positive-and Negative-Sequence Vectors from Original Unbalanced System of Vectors.

To obtain the positive-sequence system, solution of equation (7) is performed graphically. For ready reference, equation (7) is repeated:

$$V_{a1} = 1/3\ (V_a + a\,V_b + a^2\,V_c)$$

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[2] Wagner, C. F., and Evans, R. D., op. cit.

In accordance with the formula, vector V_b is rotated counterclockwise through 120 degrees as in Figure 6 above. In a similar manner, V_c is rotated through 240 degrees in the counterclockwise (or 120 degrees clockwise) direction. The vector $3V_{a1}$ is then drawn from the end points of the resolved vectors V_b and V_c. Both magnitude and phase of the vector V_{a1} may be taken from this construction.

Solution for the negative-sequence system is obtained in a like manner by performing the indicated operations of equation (8) graphically. Equation (8) is repeated:

$$V_{a2} = 1/3\ (V_a + a^2 V_b + a\ V_c)$$

This method is straightforward and, as mentioned previously, is both speedy and accurate. Using a scale of one volt per millimeter, the error introduced by this method is found to be negligible when voltages of the order of those used in the laboratory tests performed for this thesis are considered.

APPENDIX II

DETERMINATION OF THE MACHINE CONSTANTS FOR THE EQUIVALENT CIRCUIT OF A 5-HORSEPOWER, CAGE ROTOR, THREE-PHASE INDUCTION MOTOR[1]

A. Calculation of Rotor-Resistance From Design Data

The operating characteristics of the induction motor are determined to a large measure by the rotor resistance, therefore, it is mandatory that the rotor resistance must be known with as high a degree of accuracy as is possible to measure the rotor resistance directly. The design data for the rotor was obtained from the manufacturers,[2] and the rotor resistance was then calculated from the design specifications.

Number rotor slots, N_2 . . .	29
Size of rotor slot	0.145 square inches
Size of rotor conductor, A_r.	184750 circular mills
Rotor diameter	5.97 inches
Axial rotor length	3.2812 inches
Rotor wiring connection . .	Single cage
Area of end ring, A_r	1.125 " x 0.5" (715,750 circular mills)
Diameter of end ring, D_r . .	5.938 inches
Resistivity of rotor conductor material, K_b .	2.0 (compared with copper)
Length of conductor, L_2 . .	3.4 inches (skin length)
Slot pitches of skew	1.353
Number of poles for which stator is wound, P . . .	4
Number of stator phases, M .	3
Pitch factor of stator winding, K_p	0.935

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[1] The machine tested was new and of late design.

[2] The Louis Allis Company, Milwaukee, Wisconsin.

Distribution factor of
stator winding, k_d 0.960
Number of stator slots 36
Total number of stator
inductions per slot 20
Number of stator inductors
per phase, Z_1 240

$$R_2 = N_2^2\left(\frac{L_2\ k_b}{N_2 A_2} + 2\ \frac{D_r\ k_r}{\pi\ P^2 A_r}\right) \text{ ohms (total resistance of cage rotor)}$$

$$R_2 = (29)^2\left[\frac{(3.40)(2)}{29(184750)} + \frac{2(4.8125)(2)}{\pi\ (4)^2(715{,}750)}\right]$$

$R_2 = 0.001519$ ohms (corrected for skew)

Angle of skew of rotor in degrees,

$$= \frac{\text{Slot pitches of skew}}{\text{Total number rotor slots}} \times \frac{\text{Poles}}{2} \times 360$$

$$= \frac{1.353}{29} \times \frac{4}{2} \times 360$$

$= 33.6$ degrees.

Skew factor, k

$$k = \text{Sin}\ \frac{\alpha}{2} \div \frac{\alpha\,(\text{in radions})}{2}$$

$$k = \text{Sin}\ \frac{33.6}{2} \div \frac{0.5874}{2}$$

$k = 0.985$

Resistance (total) of rotor referred into the stator,

$$R_2\ (\text{in stator terms}) = \left[\frac{L_2 K_b}{A_2 N_2} + \frac{2\ D_r K_r}{\pi\ P^2 A_r}\right]\left[\frac{M Z_1 k_p k_d}{k}\right]^2$$

where $\frac{MZ_1 k_p k_d}{k}$ is the turns ratio of the machine.

$$R_2 = \left[\frac{(3.40)(2)}{29(184750)} + \frac{2(4.8125)(2)}{\pi(4)^2(715,750)}\right]\left[\frac{3(240)(0.935)(0.96)}{0.985}\right]^2$$

$R_2 = 0.785$ ohms (total).

Assuming that the rotor resistance per phase is one-third the total resistance,

$R_2 = 1/3(0.785) = 0.2617$ ohms per phase at 75° C.

This method of computing the rotor resistance from design data is due to Puchstein and Lloyd.[3]

B. Determination of the Stator Resistance, Stator Reactance, and Rotor Reactance From the No-Load and Blocked-Rotor Tests

The no-load test is made by applying a balanced system of potentials at rated frequency to the machine terminals and measuring the current and power input at rated voltage. In order that the windage and friction losses may be determined, the applied voltage is reduced in steps until the machine falls approximately five revolutions per minute below synchronous speed. The current and power input is recorded at each point and a curve of "power input versus terminal voltage" is made as in Figure K. The curve is extended until it intersects the power input axis. At this point the power input is taken to represent the friction and windage loss of the machine. The friction and windage loss for this particular motor is found to be 118 watts.

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[3] Puchstein, A. F. and Lloyd, T. C., Alternating-Current Machines, (John Wiley and Sons, Inc., 1942), pp 282-291.

In making the blocked-rotor test, the rotor is held firmly and a balanced system of voltages is applied to the stator terminals. The terminal voltage, line current and power input are measured for several values of applied potential and a plot of line current versus applied voltage is made. To insure that the current drawn by the motor not be excessive, a reduced voltage must, of necessity, be applied. The current and power input curves are then predicted for rated applied voltage in a manner suggested by Puchstein and Lloyd.[4] Figure L is a plot of the blocked-rotor conditions for this particular machine.

The equivalent impedance of the machine, by making use of the blocked-rotor test, is given by

$$Z_e = \frac{\text{rated volts per phase}}{\text{short-circuit current}}$$

$$Z_e = \frac{127}{65}$$

$$Z_e = 1.955 \text{ ohms per phase.}$$

The equivalent resistance of the rotor is determined by

$$R_e = \frac{\text{in-phase current (from Fig. L)}}{\text{total current}} \times Z_e$$

$$R_e = \frac{26.2}{65} \times 1.955$$

$$R_e = 0.7890 \text{ ohms per phase.}$$

The equivalent resistance of the machine is

$$X_e = \sqrt{Z_e^2 - R_e^2}$$

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[4]Puchstein, A. F. and Lloyd, T.C., op. cit., pp 259-263.

$$X_e = \sqrt{(1.955)^2 - (0.789)^2}$$

$$X_e = 1.789 \text{ ohms per phase.}$$

The stator leakage reactance is assumed to be equal to the rotor leakage reactance, therefore

$$X_1 = X_2 = \tfrac{1}{2} X_e = 0.8945 \text{ ohm per phase.}$$

The ohmic resistance of the stator (determined from measurement at 25°C.) is found to be 0.322 ohms per phase.

The ohmic resistance of the stator corrected to 75° C is

$$\frac{R_{75^\circ C}}{R_{25^\circ C}} = \frac{T_0 + 75}{T_0 + 25}$$

$$R_{75^\circ C} = 0.322 \left[\frac{234.5 + 75}{234.5 + 25} \right]$$

$$R_{1(75^\circ C)} = 0.384 \text{ ohm per phase.}$$

The ohmic resistance of the rotor (referred into stator terms) is

$$\frac{R_{25^\circ C}}{R_{75^\circ C}} = \frac{T_0' + 25}{T_0' + 75}$$

$$R_{2(25^\circ C)} = 0.2617 \left[\frac{236.4 + 25}{236.4 + 75} \right]$$

$$R_{2(25^\circ C)} = 0.2195 \text{ ohm per phase.}$$

The effective resistance of the rotor is now determined from a method suggested by Lawrance.[5]

$$R_{1(25^\circ C)} \text{ (effective)} = 0.785 \; \frac{0.322}{0.322 + 0.2915}$$

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[5] Lawrance, R. R., Principles of Alternating-Current Machinery, (McGraw-Hill Book Company, 1940), pp 541-543.

$R_{1(25^{o}C)} = 0.468$ ohm per phase (effective).

$R_{1(75^{o}C)} = 0.468 - 0.322 + 0.384$

$R_{1(75^{o}C)} = 0.53$ ohm per phase (effective).

Since the rotor frequency at low values of slip is quite low, the resistance of the rotor (referred into stator terms(is taken as the ohmic resistance of the rotor at 75°C., viz., 0.2617 ohms per phase.

C. Determination of the Magnetizing Branch Parameters

From the no-load tests, the power input at rated voltage is consumed by the friction and windage loss, a small stator copper loss, and the core loss.

The no-load tests are now reproduced,

$V_{(terminal)} = 220$ volts , $I_{(line)} = 5.47$ amperes , $P_{(total)} = 260$ watts.

The friction and windage loss has been previously determined to be 118 watts.

The total core loss is then

Core loss = Power input - Friction & Windage loss
- total no-load stator copper loss.

Core loss = $260 - 118 - 3(5.47)^2\ 0.53$ watts.

Core loss = 94.41 watts.

The voltage across the magnetizing branch of the equivalent circuit is

$$E_1 = V_1 - I_1 Z_1$$

$$E_1 = 127.0 - 5.47(0.8945)$$

$$E_1 = 122.1 \text{ volts per phase.}$$

Since the core loss varies as the square of the voltage, the core loss is now corrected to the voltage actually appearing across the magnetizing branch.

$$\text{Corrected core loss} = 1/3(94.41)\left[\frac{122.1}{127.0}\right]^2$$

$$\text{Corrected core loss} = 29.1 \text{ watts per phase.}$$

$$g_0 = \frac{\text{Core Loss}}{(\text{Voltage})^2} \text{ mho per phase}$$

$$g_0 = \frac{29.1}{(122.1)^2}$$

$$g_0 = 0.00195 \text{ mho per phase.}$$

The no-load current is made up of two components, namely, the hysteresis and eddy current component, I_{h+e}, and the magnetizing component, I_φ.

$$I_{h+e} = \frac{\text{core loss}}{\text{voltage}}$$

$$I_{h+e} = \frac{29.1}{122.1} = 0.2382 \text{ amperes}$$

The no-load power factor is given by

$$\cos\theta_0 = \frac{260}{3 \times 220 \times 5.47} = 0.1246$$

The magnetizing component of the no-load current is now determined by

$$I_\varphi = I\sqrt{1 - (\cos\theta_0)^2} \text{ amperes}$$

$$I = 5.47\sqrt{1 - (0.1246)^2}$$

$$I = 4.995 \text{ amperes.}$$

The magnitude of b_0 may now be calculated,

$$b_0 = \frac{I_\varphi}{E_1} \text{ mho per phase}$$

$$b_0 = \frac{4.995}{122.1} = 0.04085 \text{ mho per phase.}$$

The machine constants for the equivalent circuit diagram have now been determined and appear below in tabulated form.

Stator effective resistance (at 75°C), R_1	0.530	ohm/phase
Stator leakage reactance, X_1	0.8945	ohm/phase
Rotor leakage reactance, X_2	0.8945	ohm/phase
Rotor resistance (ohms at 75°C), R_2	0.2617	ohm/phase
Rotor resistance (120 c.p.s. at 75°C), R_2'	0.471	ohm/phase[6]
Conductance of magnetizing branch, g_0	0.00195	mho/phase
Susceptance of magnetizing branch, b_0	0.04085	mho/phase

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[6] The rotor resistance at double frequency is taken as 1.8 times the ohmic resistance of the rotor at 75°C. This is a mean value of the correction values given by several references consulted:

Lawrance, R.R., op. cit., pp 579-582. Lyon, W. V., Application of Method of Symmetrical Components, pp 295-297. Dahl, O.G.C., Electric Circuits, Vol. I, pp 138-162. Puchstein, A.F. and Lloyd, T.C. op. cit., p 33.

APPENDIX III

CALCULATION OF PERFORMANCE OF A 5-HORSEPOWER, CAGE ROTOR, THREE-PHASE INDUCTION MOTOR WITH BALANCED APPLIED VOLTAGES

Terminal voltage = 220 volts

R_1 = 0.53 ohms/phase

R_2 = 0.2617 ohms/phase

$X_1 - X_2$ = 0.8945 ohms/phase

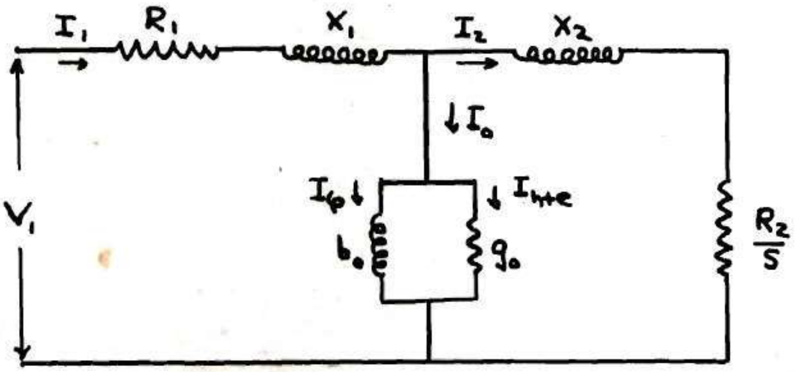
Slip = 0.0278

g_0 = 0.00195 mho/phase

b_0 = 0.04085 mho/phase

Core loss = 87.3

Friction & Windage loss = 118 watts



$$Z_2 = \frac{R_2}{S} + jX_2$$

$$Z_2 = \frac{0.2617}{0.0278} + j\,0.8945$$

$$Z_2 = 9.41 + j\,0.8945 = 9.455 \underline{|\,5.45^\circ} \text{ ohms/phase}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{9.455 \quad 5.45^\circ} = 0.1058 \underline{|\,-545^\circ} \text{ mho/phase}$$

$$Y_2 = 0.1052 - j\,0.01004 \text{ mho/phase}$$

$Y_0 = g_0 - jb_0 = 0.00195 - j\,0.04085$ mho/phase

$Y' = Y_2 + Y_0 = 0.10715 - j\,0.05089$ mho/phase

$Y' = 0.1187 \angle -25.38^\circ$ mho/phase

$$Z' = \frac{1}{Y'} = \frac{1}{0.1187 \angle -25.38^\circ} = 8.42 \angle 25.38^\circ$$

$= 7.61 + j\,3.61$ ohms/phase

$Z_t = Z' + Z_1 = 8.14 + j\,4.5045 = 9.31 \angle 28.95^\circ$ ohms/phase

$$I_1 = \frac{V_1}{Z_t} = \frac{127}{9.31 \angle 28.95^\circ} = 13.65 \angle -28.95^\circ \text{ amperes}$$

$P_1 = V_1 I_1 \cos \theta_1 = (127)(13.65) \cos 28.95^\circ$

$P_1 = 1505$ watts/phase

$P_2 = P_1 - I_1^2 R_1 - 1/3$ total core loss

$P_2 = 1505 - (13.65)^2\, 0.53 - 29.1$

$P_2 = 1379.1$ watts/phase

$P_d = P_2 (1 - S) = 1379.1 (1 - 0.0278) = 1340$ watts/phase

$P_p = P_d - 1/3$ (total friction + windage loss) $= 1340 - 39.33$

$= 1300.66$ watts/phase

Brake - horsepower $= 3 \times 1300.66/746 = 5.226$ horsepower

Efficiency $= P_p \times 100/P_1 = 1300.66 \times 100/1505 = 86.45$ per cent

$$\text{Pulley torque} = \frac{\text{Bhp} \times 33000}{2 \times \text{RPM}} = \frac{5.226 \times 33000}{2 \times 1750}$$

$= 15.71$ pounds feet

APPENDIX IV

CALCULATION OF PERFORMANCE OF A 5-HORSEPOWER, CAGE ROTOR, THREE-PHASE INDUCTION MOTOR WITH UNBALANCED APPLIED VOLTAGES

Voltage System applied to stator terminals: 242 - 22 1 - 206

$V_1 = 128.1$ volts/phase

$V_1^- = 12.71$ volts/phase

$R_1 = 0.53$ ohms/phase

$R_2 = 0.2617$ ohms/phase

$R_2' = 0.471$ ohms/phase

$X_1 = X_2 = 0.8945$ ohms/phase

Slip $= 0.0139$

$g_0 = 0.00195$ mho/phase

$b_0 = 0.04085$ mho/phase

Friction + windage loss = 118 watts

The equivalent circuit diagrams for the positive- and negative-sequence systems are given in Figure 5, Page 17.

$$Z^+ = R_1 + jX_1 + \frac{(R_2/S + jX_2)(R_0 + jX_0)}{(R_2/S + jX_2) + (R_0 + jX_0)}$$

$$Z^+ = 0.53 + j\,0.8945 + \frac{(0.2617/0.0139 + j\,0.8945)(1.16 + j\,24.4)}{(0.2617/0.0139 + j\,0.8945) + (1.16 + j\,24.4)}$$

$$Z^{+} = 11.75 + j\,9.7545 = 15.26 \angle 39.7^{\circ} \quad \text{ohms/phase}$$

$$Z^{-} = R_1 + jX_1 + \frac{(R_2'/(2 - S) + jX_2)(R_0 + jX_0)}{(R_2'/(2 - S) + jX_2) + (R_0 + jX_0)}$$

$$Z^{-} = 0.53 + j\,0.8945 + \frac{(0.471/(2 - 0.0139) + j\,0.8945)(1.16 + j\,24.4)}{(0.471/(2 - 0.0139) + j\,0.8945) + (1.16 + j\,24.4)}$$

$$Z^{-} = 0.743 + j\,1.7595 = 1.91 \angle 67.1^{\circ} \quad \text{ohms/phase}$$

$$\left|I_1^{+}\right| = \frac{\left|V_1^{+}\right|}{\left|Z^{+}\right|} = \frac{128.1}{15.26} = 8.40 \text{ amperes}$$

$$\left|I_1^{-}\right| = \frac{\left|V_1^{-}\right|}{\left|Z^{-}\right|} = \frac{12.71}{1.91} = 6.65 \text{ amperes}$$

$$\left|I_2^{+}\right| = \left|I_1^{+}\right| \frac{\left|Z_m\right|}{\left|Z_m\right| + \left|Z_2^{+}\right|} = 8.40 \; \frac{24.45}{32.22} = 6.38 \text{ amperes}$$

$$\left|I_2^{-}\right| = \left|I_1^{-}\right| \frac{\left|Z_m\right|}{\left|Z_m\right| + \left|Z_2^{-}\right|} = 6.65 \; \frac{24.45}{25.31} = 6.43 \text{ amperes}$$

$$T^{+} = \frac{3\,P}{4\pi f_1}\left[(I_2^{+})^2 \;\frac{R_2}{S}\right]\frac{550}{746}$$

$$T^{+} = \frac{3\,(4)}{2\,(377)}\left[(6.38)^2 \; 18.82\right]\frac{550}{746} = 8.98 \text{ pounds feet.}$$

$$T^{-} = \frac{3P}{4\pi f_1}\left[(I_2^{-})^2 \frac{R_2'}{2 - S}\right] \frac{550}{746}$$

$$T^{-} = \frac{3\,(4)}{2\,(377)}\left[(6.43)^2 \;\; 0.2378\right] \frac{550}{746} = 0.1152 \text{ pounds feet}$$

$$T = T - T^{-} = 8.98 - 0.1152 = 8.8648 \quad \text{pounds feet}$$

$$P_p = 2 \times \frac{2f_1}{P}\,(1 - S)\,T - P_{f+w}$$

$$P_p = 2 \times \frac{2(60)}{4}\,(1 - 0.0139)\,8.8648 - (118)\,\frac{550}{746}$$

P_p = 1555 foot pounds/second

Horsepower output = 1555/550 = 2.83 horsepower

Copper loss:

$$C_L^+ = 3\left[(I_1^+)^2 R_1 + (I_2^+)^2 R_2\right]$$

$$C_L^+ = 3\left[(8.4)^2\, 0.53 + (6.38)^2\, 0.2617\right] = 144.18 \text{ watts}$$

$$C_L^- = 3\left[(I_1^-)^2 R_1 + (I_2^-)^2 R_2'\right]$$

$$C_L^- = 3\left[(6.65)^2\, 0.53 + (6.43)^2\, 0.471\right] = 128.76 \text{ watts}$$

Copper loss $= C_L^+ + C_L^- = 144.18 + 128.76 = 272.94$ watts

Core loss:

$$\text{Stator core loss} = 3\left[|I_2^+|\,|Z_2^+| + |I_2^-|\,|Z_2^-|\right]^2 g_0$$

$$= 3\left[6.65(18.87) + 6.43(0.9249)\right]^2 0.00195$$

$$= 84.8$$

Total core loss = 84.8 x 1.045 = 88.6 watts

Total losses = copper loss + core loss + friction and windage loss

= 272.94 + 88.6 + 118 = 479.54 watts

$$\text{Efficiency} = \frac{P_p \times 100}{P_p + \text{losses}} = \frac{1555 \times 100}{1555 + 479.54} = 81.5 \text{ per cent}$$

APPENDIX V

CALCULATION OF PERFORMANCE OF A

5-HORSEPOWER, CAGE ROTOR, THREE-PHASE

INDUCTION MOTOR OPERATING SINGLE PHASE

Applied voltage = 220 volts

$R_1 = 0.53$ ohms/phase

$R_2 = 0.2617$ ohms/phase

$R_2' = 0.471$ ohms/phase

$X_1 = X_2 = 0.8945$ ohms/phase

Slip = 0.00555

$g_0 = 0.00195$ mho/phase

$b_0 = 0.04085$ mho/phase

friction + windage loss = 118 watts

$$Z^+ + Z^- = \left(2R_1 + \frac{R_2}{S} + \frac{R_2'}{2 - S}\right) + j\,2\,(X_1 + X_2)$$

$$Z^+ + Z^- = (1.06 + 47.1 + 0.2361) + j\,2(1.789)$$

$$Z^+ + Z^- = 48.3961 + j\,3.578 = 48.5 \angle 4.24^\circ \text{ ohms/phase}$$

$$Z_n = \left(2R_1 + \frac{R_2'}{2 - S}\right) + j\,(2X_1 + X_2)$$

$$Z_n = (1.06 + 0.2361) + j\,(1.789 + 0.8945)$$

$$Z_n = 1.2961 + j\,2.6835 = 2.984 \angle 64.24^\circ \text{ ohms/phase}$$

$$V^+ = \frac{V_1}{\sqrt{3}} - \frac{I_0}{\sqrt{3}}\left(\frac{R_2'}{2 - S}\right) = 127.1 - \frac{8.4}{3}\,(0.2361)$$

$$= 112.65 \text{ volts/phase}$$

$$I_2^+ = \frac{V^+}{Z^+ + Z^-} = \frac{112.65}{48.5} = 2.322 \text{ amperes}$$

$$\text{No-load power factor} = \frac{\text{no-load power input}}{\text{terminal voltage} \times \text{current}}$$

$$\text{Cos } \theta_0 = \frac{315}{220 \times 8.4} = 0.1705 \quad ; \quad \text{Sin } \theta_0 = 0.986$$

$$\text{Core loss} = \text{no-load power input} - P_{f+w} - (I_0)^2 \; (2R_1 + R_2')$$

$$= 315 - 118 - (8.4)^2 \left(2\,(0.53) + 0.471\right)$$

$$= 89 \text{ watts.}$$

$$I_n^+ = \frac{1}{\sqrt{3}} \left[- \frac{\text{core loss}}{\text{voltage}} + j\, I_0 \text{ Sin } \theta_0 \right]$$

$$= \frac{1}{\sqrt{3}} \left[- \frac{89}{220} + j\,(8.4)(0.986) \right]$$

$$= -0.23333 + j\, 4.78 \text{ amperes}$$

$$\text{Cos } \alpha^+ = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}} = \frac{0.2617}{\sqrt{(0.2617)^2 + [(0.00555)(0.8945)]^2}}$$

$$= 0.9985$$

$$\text{Sin } \alpha^+ = \frac{SX_2}{\sqrt{R_2^2 + (SX_2)^2}} = \frac{(0.00555)(0.8945)}{\sqrt{(0.2617)^2 + (0.00555\,(0.8945))^2}}$$

$$= 0.01898$$

$$I_1^+ = I_1 + I_n^+ = -I_2^+ (\text{Cos } \alpha^+ - j \text{ Sin } \alpha^+) + I_n^+$$

$$= -2.322\,(0.9985 - j0.0474) + (-\,0.2333 + j\, 4.78)$$

$$= -2.5533 + j\, 4.834 = -\,5.487 \,\underline{\big|\, -\,62.25} \text{ amperes}$$

Torque developed

$$T = \frac{3P}{4\pi f_1} \left[(I_2^+)^2 \frac{R_2}{S} - (I_2^-)^2 \frac{R_2'}{2 - S} \right] \frac{550}{746}$$

$$T = \frac{3\,(4)}{2\,(377)} \left[(2.322)^2\, 47.1 - (5.487)^2\, 0.2361 \right] \frac{550}{746}$$

$$T = 2.898 \text{ pounds feet}$$

Pulley power

$$P_p = 2\pi \times \frac{2f_1}{P} (1 - S)\ T - P_{f+w}$$

$$P_p = 2\pi \times \frac{2\ (60)}{4} (1 - 0.00555)\ 2.898 - 118\ \frac{550}{746}$$

$$P_p = 542.5 - 87 = 455.5 \text{ foot pounds/second.}$$

Horsepower output $= 455.5/550 = 0.8255$ horsepower

Line current $= \sqrt{3}\ I_1^{+} = \sqrt{3}\ (5.487) = 9.5$ amperes

$$\begin{aligned} \text{Copper loss} &= 3\left[2\ (I_1^{+})^2\ R_1 + (I_2^{+})^2\ R_2 + (I_1^{+})^2\ R_2'\right] \\ &= 3\left[2\ (5.487)^2\ 0.53 + (2.322\ )^2\ 0.2617 + (5.487)^2\ (0.471)\right] \\ &= 129.83 \text{ watts.} \end{aligned}$$

$$\begin{aligned} \text{Total losses} &= \text{copper loss} + \text{core loss} + \text{friction and windage loss} \\ &= 129.83 + 89 + 118 \\ &= 336.83 \text{ watts.} \end{aligned}$$

$$\text{Efficiency} = \frac{P_p \times 100}{P_p + \text{losses}} = \frac{618 \times 100}{618 + 336.83} = 64.9 \text{ per cent.}$$

TABLE I

MEASURED CHARACTERISTICS WITH BALANCED APPLIED VOLTAGES
APPLIED VOLTAGES 220-220-220

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
5.60	5.25	5.65	260	----	1800	---	---	---	---
6.40	6.35	6.45	1380	4.575	1785	4.71	1.60	86.5	0.834
8.10	8.10	8.25	2280	6.350	1775	8.26	2.76	90.4	1.390
10.00	10.05	10.03	3060	7.667	1765	10.90	3.66	89.2	1.946
13.45	13.60	13.65	4340	9.860	1750	15.20	5.06	87.3	2.780
16.60	16.70	16.80	5500	11.900	1735	19.36	6.39	86.7	3.610

TABLE II

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 242-221-206

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PERCENT	SLIP PER CENT
4.40	8.00	10.88	350	---	1800	---	---	----	----
4.04	8.80	11.10	769	2.79	1795	1.49	0.51	49.5	0.278
3.04	9.68	11.41	1180	3.82	1790	3.20	1.09	69.0	0.555
2.72	10.51	11.85	1616	4.74	1785	5.03	1.71	79.0	0.834
2.67	11.53	11.25	2125	6.10	1780	6.76	2.29	82.5	1.110
3.53	12.92	12.70	2585	6.48	1775	8.51	2.88	83.0	1.390

TABLE III

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 240-216-195

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
4.79	9.25	13.05	410	---	1800	---	---	---	-----
4.42	10.00	13.35	696	2.84	1795	1.23	0.42	45.0	0.278
3.98	10.70	13.62	1221	3.76	1790	3.08	1.05	64.1	0.555
3.73	11.95	14.05	1640	4.59	1785	4.74	1.61	73.3	0.834
3.50	13.65	14.25	2080	5.44	1780	6.44	2.18	78.2	1.110

TABLE IV

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 230-214-200

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
6.50	10.84	5.50	355	---	1800	---	---	---	----
6.75	10.30	5.20	1069	3.69	1790	2.94	1.00	69.8	0.555
8.35	11.10	4.85	1500	4.56	1785	4.68	1.59	79.0	0.834
10.28	11.88	3.52	1965	5.41	1780	6.38	2.16	82.1	1.110
11.42	11.72	3.80	2416	6.10	1775	7.96	2.69	83.1	1.390
12.50	12.60	4.10	2845	6.99	1770	9.53	3.21	84.2	1.667

TABLE V

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 230-207-191

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
5.59	8.00	11.92	375	----	1800	---	----	---	----
5.10	8.93	12.12	820	2.81	1795	1.17	0.40	36.[illegible]	0.278
4.41	10.10	12.32	1148	3.60	1790	2.82	0.96	62.5	0.555
3.51	11.32	12.55	1522	4.45	1785	4.45	1.51	74.0	0.834
3.52	12.40	12.78	1925	5.25	1780	6.05	2.05	79.5	1.110
3.83	13.66	13.01	2310	5.98	1775	7.52	2.54	82.0	1.390

TABLE VI

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 226-204-194

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
3.35	6.70	9.02	320	---	1800	---	---	---	---
3.27	7.80	9.50	1090	3.61	1790	2.79	0.95	65.0	0.555
2.35	9.60	10.10	1490	4.45	1785	4.47	1.32	76.0	0.834
2.57	12.10	11.90	1883	5.17	1780	5.90	2.00	79.2	1.110
3.63	12.50	11.20	2290	5.94	1775	7.45	2.52	82.1	1.390
4.61	13.01	11.70	2655	6.51	1770	8.79	2.96	83.2	1.667

TABLE VII

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 222-202-187

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
3.00	8.60	10.60	330	---	1800	---	---	----	----
2.82	11.75	11.70	990	3.50	1790	2.57	0.87	66.0	0.555
2.72	12.35	12.01	1473	4.42	1785	4.41	1.50	76.1	0.834
3.08	13.01	12.26	1800	5.05	1780	5.66	1.92	79.5	1.110
3.40	13.96	12.87	2220	5.83	1775	7.21	2.44	82.0	1.390
4.30	14.40	13.20	2580	6.47	1770	8.50	2.86	82.8	1.667

TABLE VIII

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 219-196-175

I_2 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
7.82	5.62	11.09	385	---	1800	---	----	----	----
9.20	5.45	11.80	618	2.69	1795	0.95	0.33	39.2	0.278
12.95	5.32	12.22	1002	3.40	1790	2.40	0.82	61.0	0.555
13.30	5.36	12.62	1404	4.16	1785	3.88	1.32	70.1	0.834
13.75	5.39	13.06	1710	4.78	1780	5.13	1.74	76.0	1.110
14.70	5.51	13.70	2122	5.55	1775	5.66	2.25	79.1	1.390

TABLE IX

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES: 217-221-194

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
4.75	3.82	6.10	315	---	1800	---	---	---	----
5.51	3.99	8.00	1178	3.86	1790	3.28	1.12	71.0	0.550
6.69	4.25	10.10	1451	4.45	1785	4.47	1.52	78.2	0.834
7.94	4.33	11.00	1931	5.34	1780	6.25	2.12	82.0	1.110
9.09	4.50	11.88	2265	5.99	1775	7.65	2.55	84.0	1.390
10.47	4.70	12.41	2680	6.73	1770	9.02	3.04	84.6	1.667

TABLE X

MEASURED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
SINGLE PHASE OPERATION

I_1 AMP.	I_2 AMP.	I_3 AMP.	TOTAL WATTS INPUT	SCALE WEIGHT POUNDS	SPEED RPM	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
8.40	--	8.40	320	----	1800	----	----	-----	----
9.30	--	9.30	1010	3.54	1790	2.64	0.90	66.5	0.555
9.72	--	9.72	1232	3.97	1785	3.50	1.19	72.0	0.834
11.40	--	11.40	1745	4.85	1780	5.26	1.78	76.2	1.110
12.75	--	12.75	2040	5.40	1775	6.36	2.16	78.9	1.390
14.60	--	14.60	2420	6.05	1770	7.66	2.58	79.6	1.667

TABLE XI

CALCULATED CHARACTERISTICS WITH BALANCED APPLIED VOLTAGES

PHASE VOLTAGE V	PHASE CURRENT I	POWER INPUT WATTS	PULLEY TORQUE LB.-FT.	HORSE-POWER OUTPUT	EFFICIENCY PER CENT	SLIP PER CENT
127.1	6.34	1528	4.91	1.67	81.6	0.834
127.1	8.33	2441	8.29	2.80	85.7	1.390
127.1	10.41	3310	11.42	3.84	86.6	1.946
127.1	13.65	4515	15.71	5.23	86.4	2.780
127.1	16.78	5675	19.61	6.48	85.2	3.610

TABLE XII

CALCULATED CHARACTERISTIC FOR SINGLE-PHASE OPERATION

APPLIED VOLTAGE V	PHASE CURRENT I	INTERNAL TORQUE LB.-FT.	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
127.1	9.50	2.90	0.83	336.8	64.9	0.555
127.1	10.50	4.07	1.23	384.7	70.5	0.834
127.1	11.83	5.44	1.69	437.0	74.1	1.110
127.1	13.23	6.58	2.07	399.1	75.6	1.390
127.1	14.56	7.69	2.43	565.2	76.3	1.667

TABLE XIII

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGE
APPLIED VOLTAGES (TERMINAL): 242-221-206
SEQUENCE VOLTAGES (PHASE): $V^{+} = 128.1$, $V^{-} = 12.41$

SEQUENCE CURRENTS (AMPERES) I^{+}	I^{-}	INTERNAL TORQUE (POUNDS-FEET) T^{+}	T^{-}	T	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
5.245	6.65	1.871	0.141	1.7669	0.442	383.5	46.3	0.278
5.760	6.65	3.715	0.1145	3.6005	1.069	396.2	66.7	0.555
6.520	6.65	5.510	0.1148	5.3952	1.675	416.4	75.1	0.834
7.400	6.65	7.225	0.1150	7.1100	2.246	442.9	79.1	1.110
8.400	6.65	8.980	0.1152	8.8648	2.830	479.5	81.5	1.390

TABLE XIV

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 240-216-195
SEQUENCE VOLTAGES (PHASE): $V^+ = 124.7$, $V^- = 15.35$

SEQUENCE CURRENTS (AMPERES) I^+	I^-	INTERNAL TORQUE (POUNDS-FEET) T^+	T^-	T	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
5.10	8.04	1.768	0.1670	1.6010	0.389	434.2	40.2	0.278
5.61	8.04	3.500	0.1673	3.3327	0.978	446.1	62.1	0.555
6.35	8.04	5.225	0.1677	5.0573	1.56	465.6	71.6	0.834
7.195	8.04	6.840	0.1680	6.6720	2.10	490.2	76.2	1.110

TABLE XV

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 230-214-200
SEQUENCE VOLTAGES (PHASE): $V^+ = 123.5$, $V^- = 10.40$

SEQUENCE CURRENTS (AMPERES)		INTERNAL TORQUE (POUNDS - FEET)			HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
I^+	I^-	T^+	T^-	T				
5.557	5.442	3,440	0.0769	3.3631	0.986	343.2	68.3	0.555
6.292	5.442	5.112	0.0770	5.0350	1.557	362.6	76.3	0.834
7.125	5.442	6.725	0.0771	6.6479	2.100	387.6	80.2	1.110
8.100	5.442	8.350	0.0772	8.2728	2.640	420.8	82.5	1.390
9.085	5.442	9.840	0.0773	9.7627	3.133	458.6	83.8	1.667

TABLE XVI

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 230-207-191
SEQUENCE VOLTAGES (PHASE): $V^{+} = 120.5$, $V^{-} = 12.88$

SEQUENCE CURRENTS (AMPERES) I^{+}	I^{-}	INTERNAL TORQUE (POUNDS-FEET) T^{+}	T^{-}	T	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
4.930	6.74	1.657	0.1175	1.5395	0.368	372.6	42.5	0.278
5.423	6.74	3.280	0.1179	3.1621	0.916	382.8	64.1	0.555
6.140	6.74	4.875	0.1181	4.7569	1.460	399.2	73.3	0.834
6.955	6.74	6.395	0.1182	6.2768	1.972	422.9	77.8	1.110
7.900	6.74	7.940	0.1183	7.8217	2.490	455.9	80.4	1.390

TABLE XVII

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 226-204-194
SEQUENCE VOLTAGES (PHASE): $V^+ = 119.9$, $V^- = 11.37$

SEQUENCE CURRENTS (AMPERES) I^+	I^-	INTERNAL TORQUE (POUNDS-FEET) T^+	T^-	T	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
5.395	5.95	3.250	0.0919	3.1581	0.915	351.8	62.4	0.555
6.105	5.95	4.838	0.0921	4.7359	1.451	370.1	74.6	0.834
6.910	5.95	6.305	0.0922	6.2128	1.915	392.9	78.4	1.110
7.852	5.95	7.845	0.0924	7.7546	2.462	424.5	81.1	1.390
8.810	5.95	9.250	0.0925	9.1575	2.935	460.3	82.7	1.667

TABLE XVIII

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 222-202-187
SEQUENCE VOLTAGES (PHASE): $V^{+} = 117.2$, $V^{-} = 11.73$

SEQUENCE CURRENTS (AMPERES)		INTERNAL TORQUE (POUNDS-FEET)			HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
I^{+}	I^{-}	T^{+}	T^{-}	T				
5.277	6.15	3.102	0.0977	3.0143	0.866	353.4	64.7	0.555
5.975	6.15	4.630	0.0979	4.5321	1.386	370.7	73.6	0.834
6.775	6.15	6.055	0.0981	5.9579	1.863	392.9	78.0	1.110
7.690	6.15	7.500	0.0983	7.4017	2.351	422.9	80.7	1.390
8.630	6.15	8.880	0.0984	8.7816	2.800	457.3	82.1	1.667

TABLE XIX

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 219-196-175
SEQUENCE VOLTAGES (PHASE): $V^{+} = 113$, $V^{-} = 14.28$

SEQUENCE CURRENTS (AMPERES)		INTERNAL TORQUE (POUNDS-FEET)			HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
I^{+}	I^{-}	T^{+}	T	T^{-}				
4.625	7.48	1.461	0.1448	1.3162	0.29	387.6	36.9	0.278
5.085	7.48	2.375	0.1450	2.7300	0.77	396.9	59.2	0.555
5.760	7.48	4.300	0.1452	4.1548	1.25	413.2	69.4	0.834
6.520	7.48	5.610	0.1456	5.4644	1.69	432.7	74.6	1.110
7.410	7.48	6.960	0.1458	6.8142	2.14	461.8	77.7	1.390

TABLE XX

CALCULATED CHARACTERISTICS WITH UNBALANCED APPLIED VOLTAGES
APPLIED VOLTAGES (TERMINAL): 217-212-194
APPLIED VOLTAGES (PHASE): $V^{+} = 119.8$, $V^{-} = 8.09$

SEQUENCE CURRENTS (AMPERES) I^{+}	I^{-}	INTERNAL TORQUE (POUNDS-FEET) T^{+}	T^{-}	T	HORSE-POWER OUTPUT	WATTS POWER LOSS	EFFICIENCY PER CENT	SLIP PER CENT
5.39	4.235	3.235	0.0464	3.1886	0.93	300.8	69.8	0.555
6.10	4.235	4.815	0.0465	4.7685	1.46	318.5	77.5	0.834
6.90	4.235	6.290	0.0466	6.2434	1.97	341.2	81.1	1.110
7.85	4.235	7.845	0.0467	7.7983	2.47	372.5	83.4	1.390
8.81	4.235	9.250	0.0467	9.2033	2.94	408.9	84.1	1.667

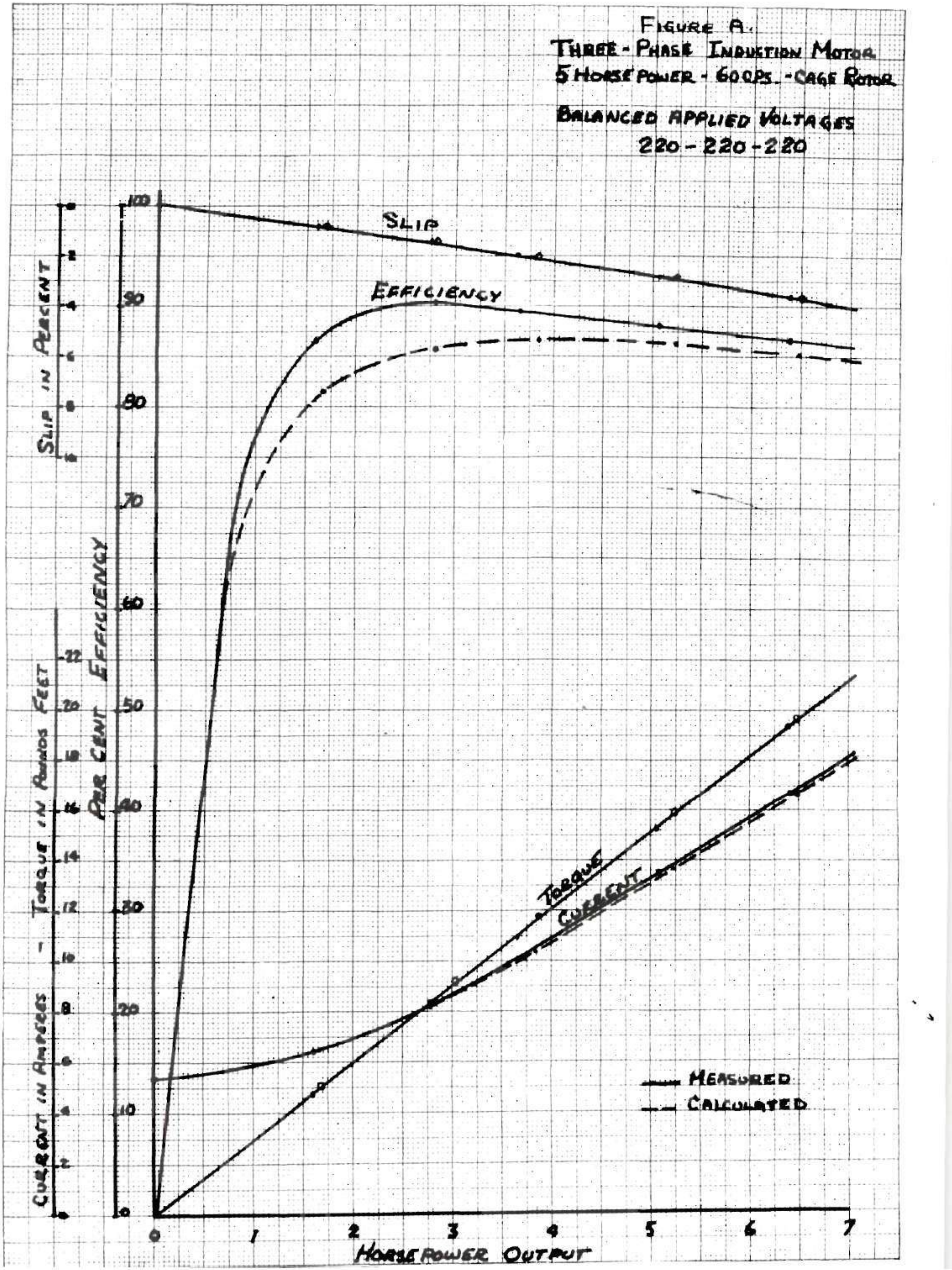


FIGURE A.
THREE - PHASE INDUCTION MOTOR
5 HORSE POWER - 60 CPS. - CAGE ROTOR
BALANCED APPLIED VOLTAGES
220 - 220 - 220
SLIP
EFFICIENCY
TORQUE
CURRENT
MEASURED
CALCULATED
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
HORSE POWER OUTPUT
0
1
2
3
4
5
6
7
100
90
80
70
60
50
40
30
20
10

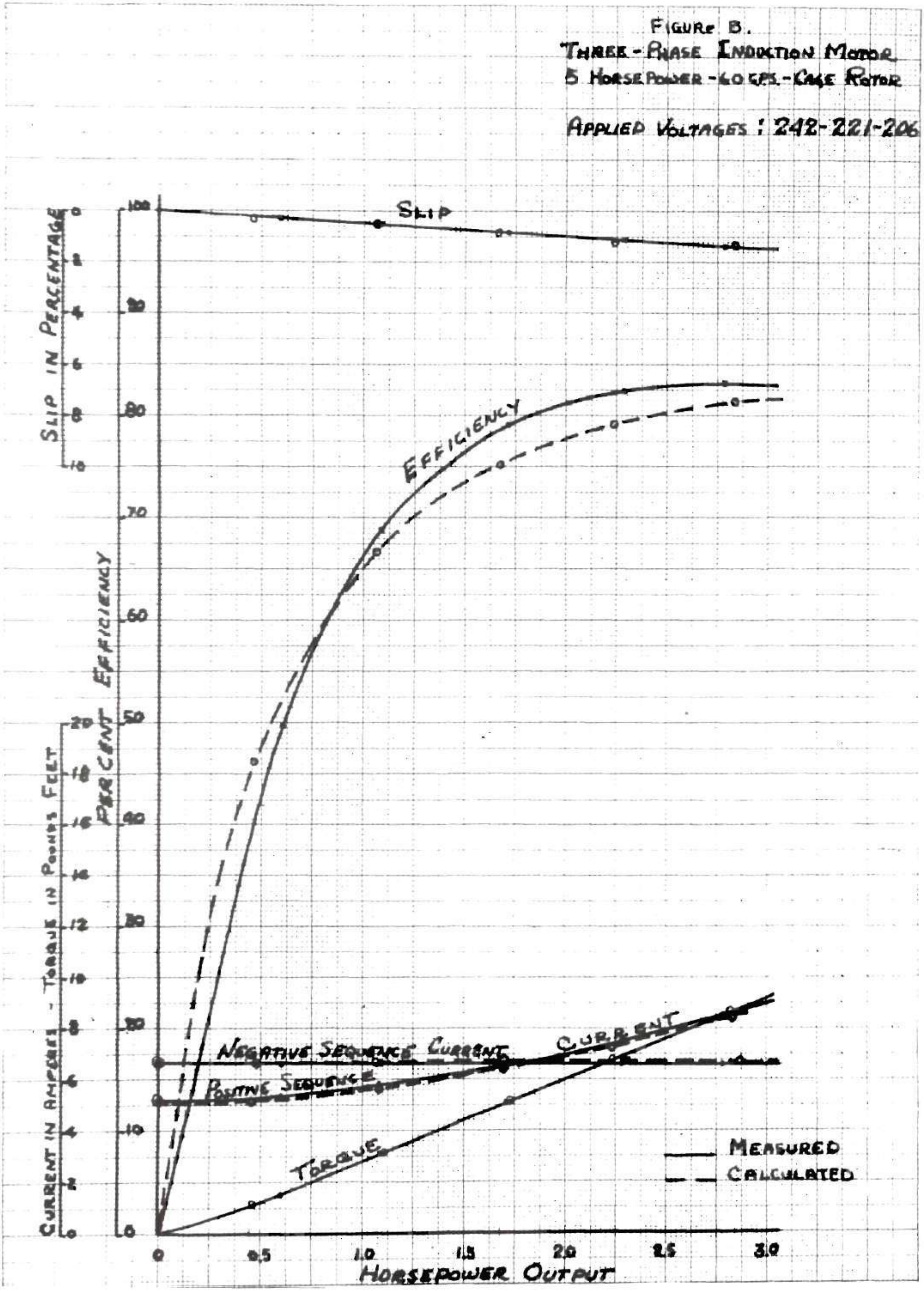


FIGURE B.
THREE - PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 CPS. - CAGE ROTOR
APPLIED VOLTAGES : 242-221-206
SLIP
EFFICIENCY
NEGATIVE SEQUENCE CURRENT
CURRENT
POSITIVE SEQUENCE
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENTAGE
PERCENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0

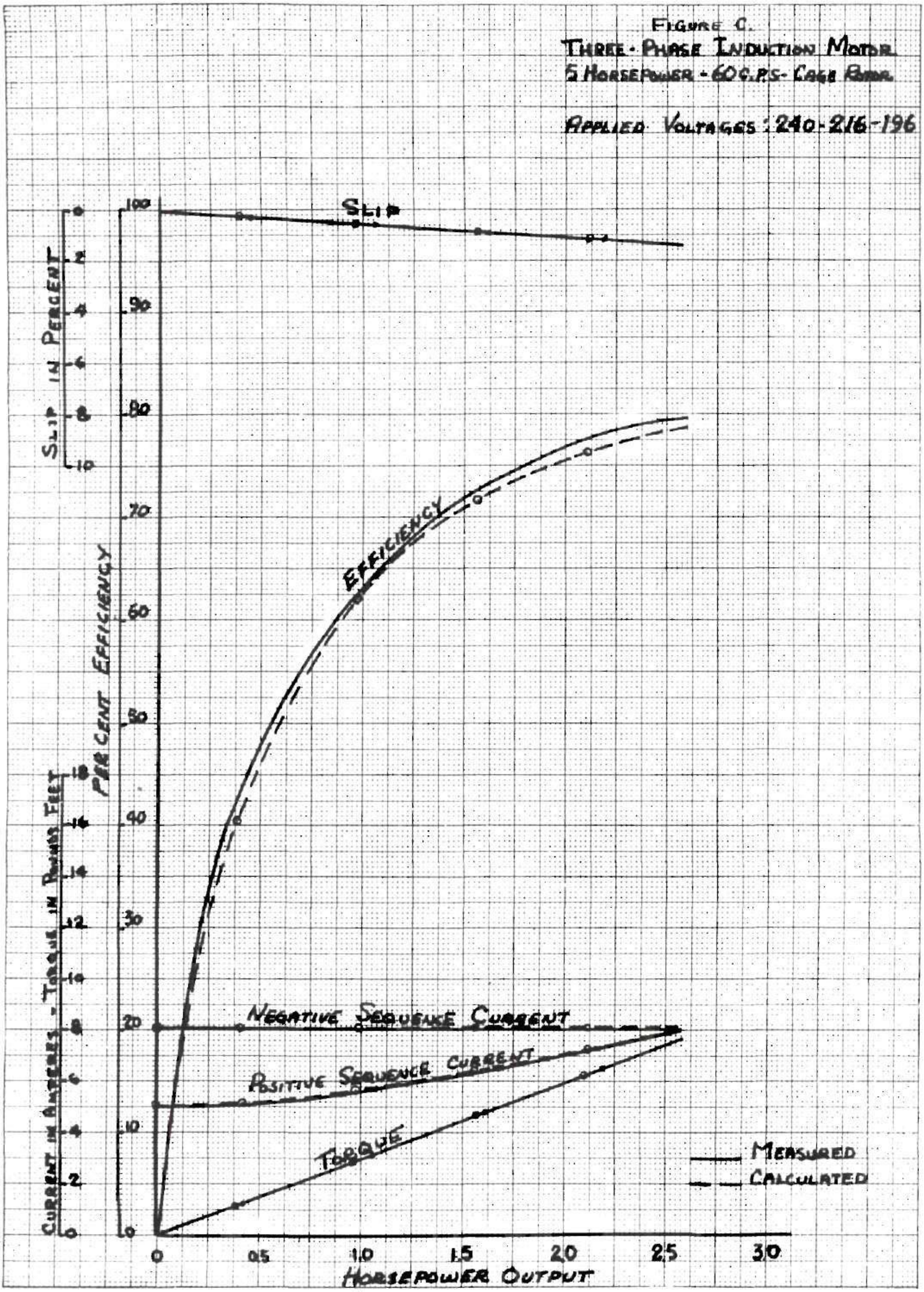


FIGURE C.
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.P.S. - CAGE ROTOR
APPLIED VOLTAGES: 240-216-196
SLIP
EFFICIENCY
NEGATIVE SEQUENCE CURRENT
POSITIVE SEQUENCE CURRENT
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUND FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0

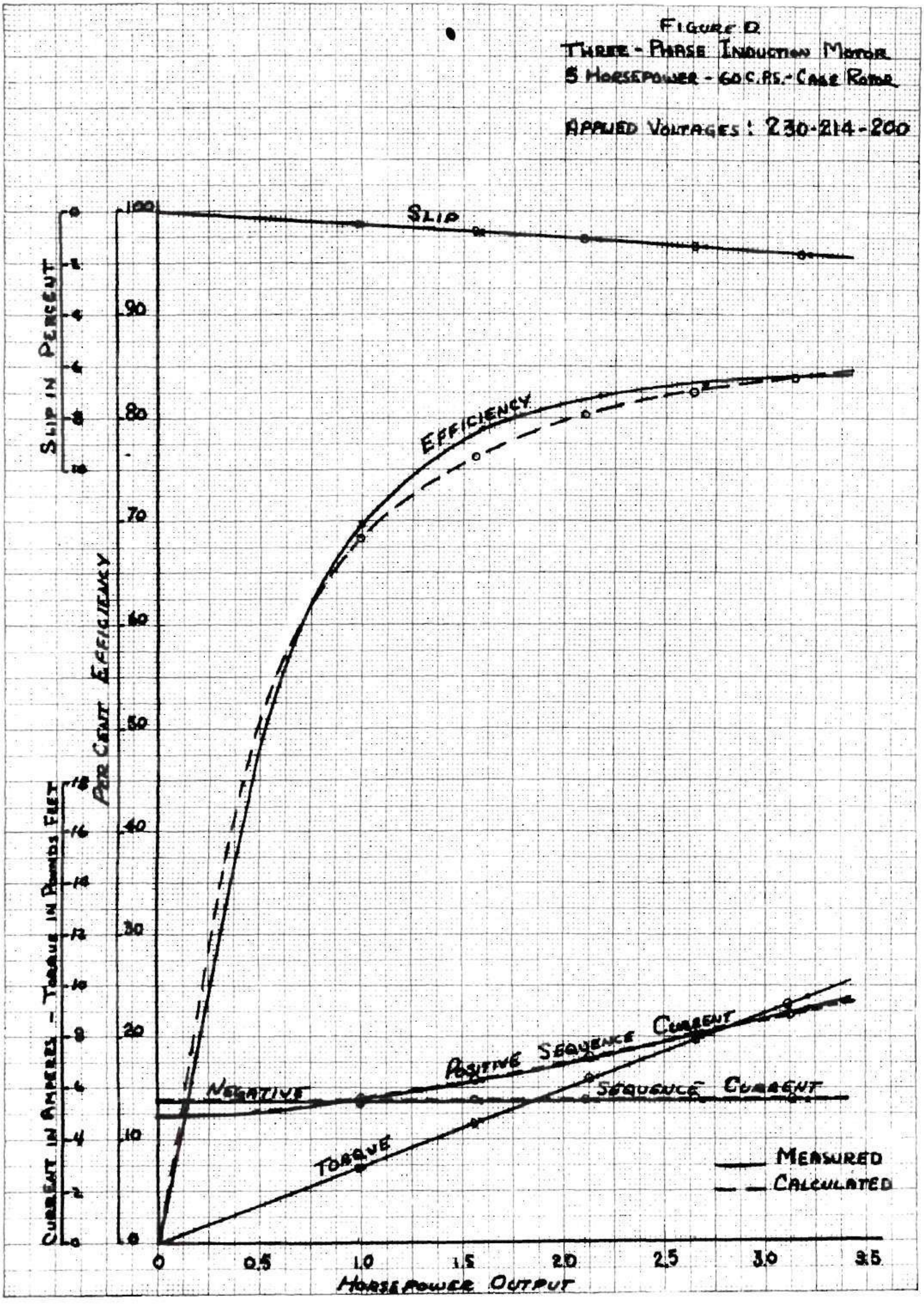
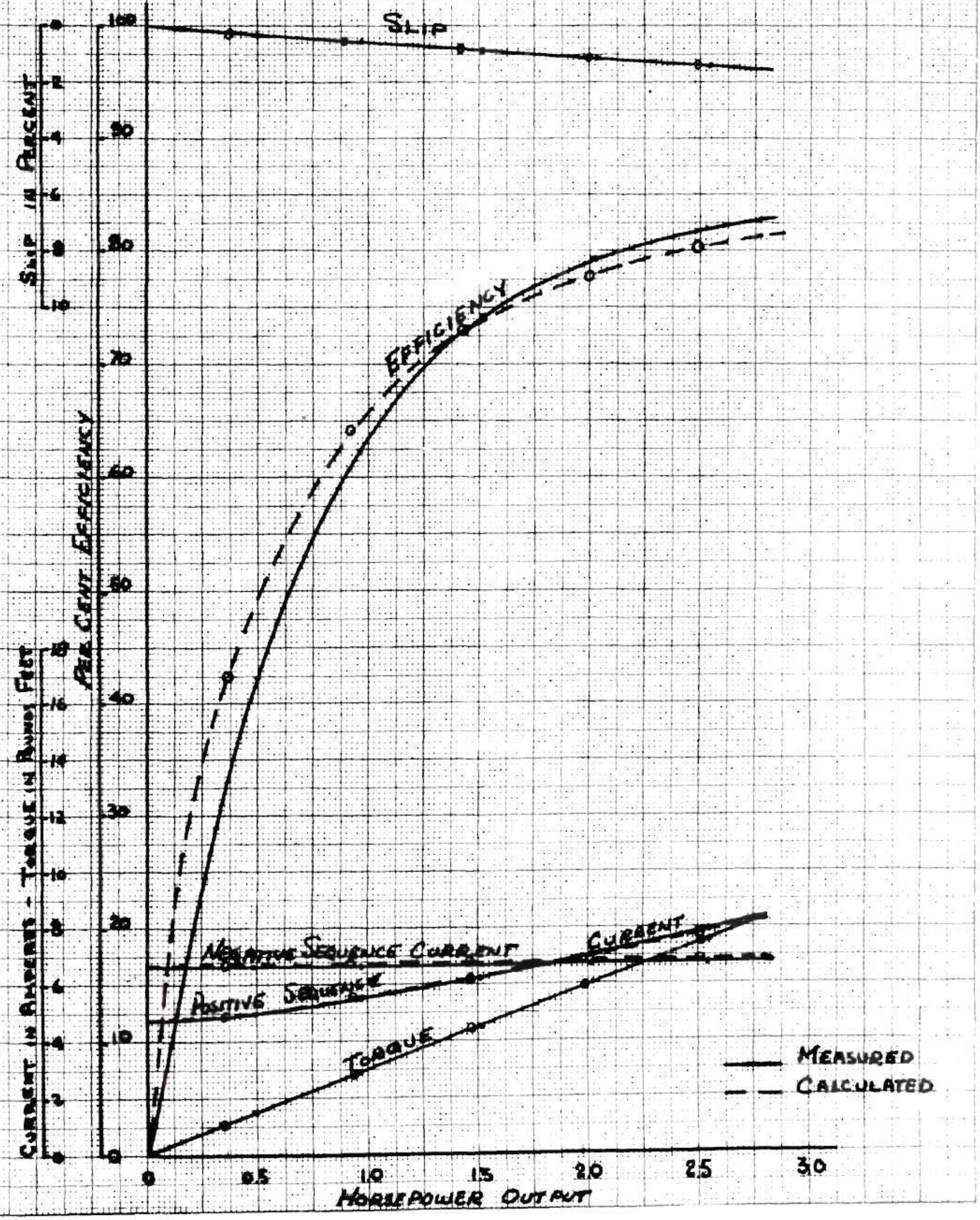


FIGURE D
THREE - PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.PS. - CAGE ROTOR
APPLIED VOLTAGES: 230-214-200
SLIP
EFFICIENCY
POSITIVE SEQUENCE CURRENT
NEGATIVE SEQUENCE CURRENT
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
100
90
80
70
60
50
40
30
20
10
0

FIGURE E.

THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.P.S. - CAGE ROTOR

APPLIED VOLTAGES : 230-207-191



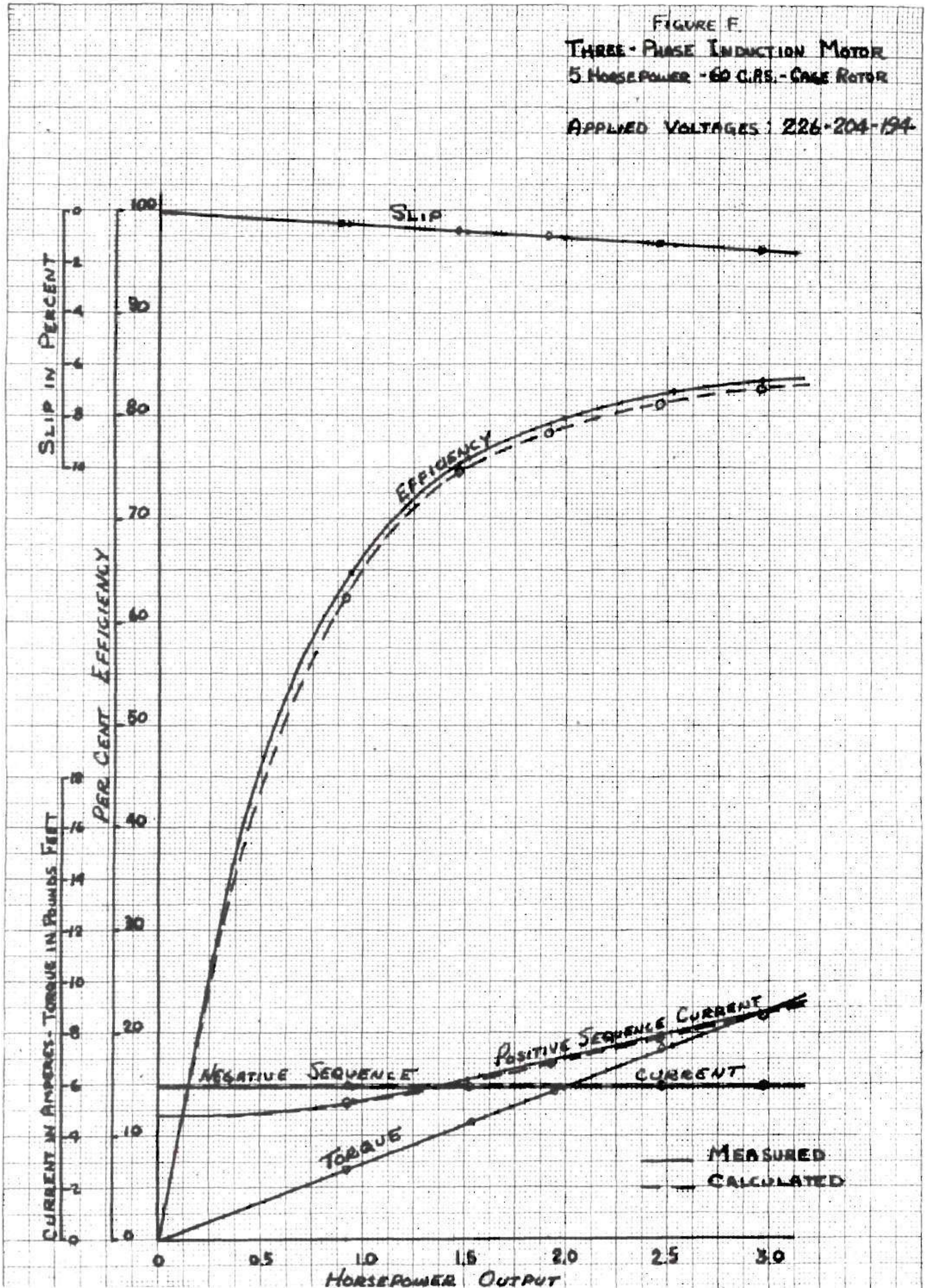


FIGURE F.
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.PS. - CAGE ROTOR
APPLIED VOLTAGES: 226-204-194
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
SLIP
EFFICIENCY
POSITIVE SEQUENCE CURRENT
NEGATIVE SEQUENCE CURRENT
TORQUE
MEASURED
CALCULATED
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0

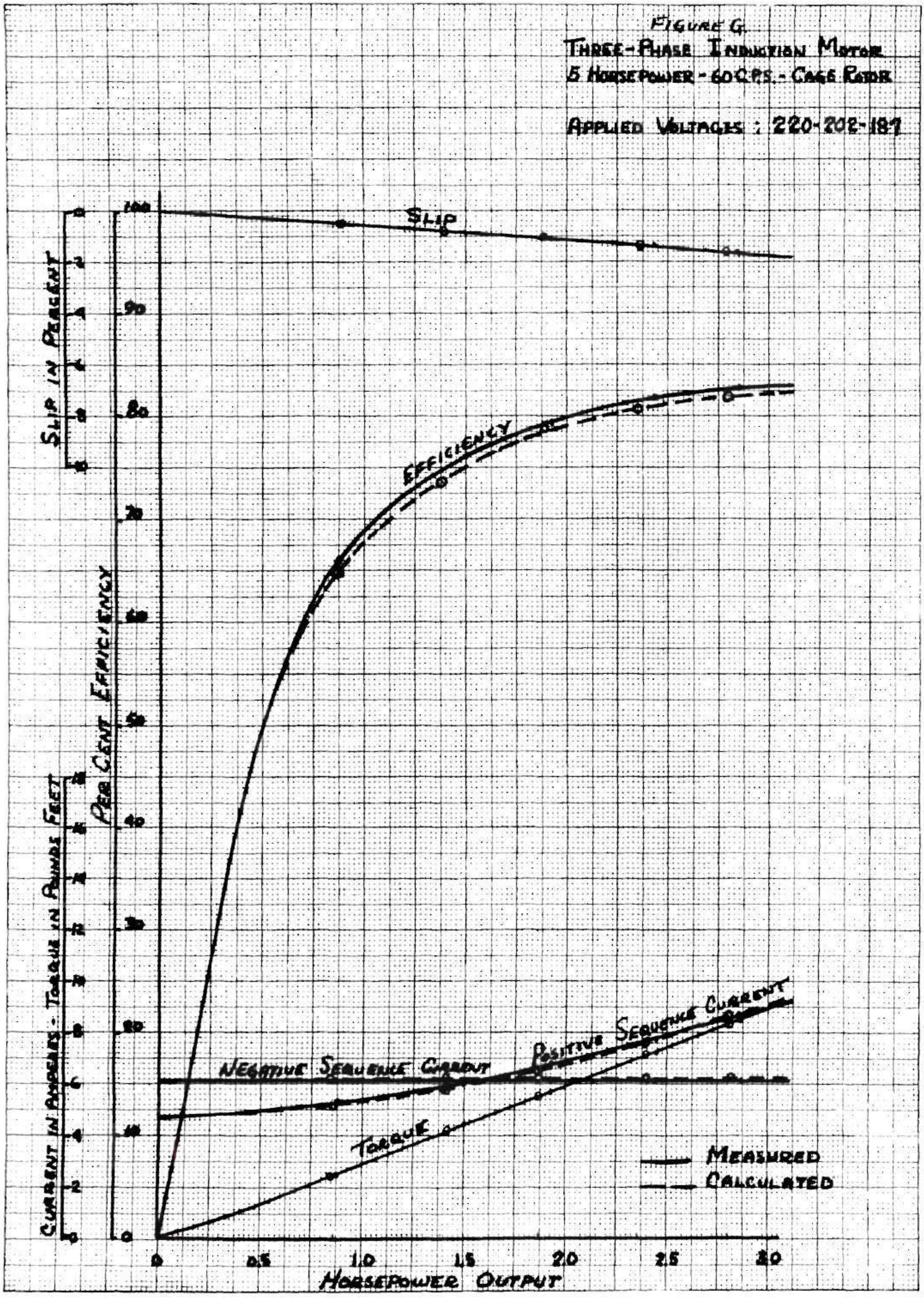


FIGURE G.
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.P.S. - CAGE ROTOR
APPLIED VOLTAGES : 220-202-187
SLIP
EFFICIENCY
NEGATIVE SEQUENCE CURRENT
POSITIVE SEQUENCE CURRENT
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0

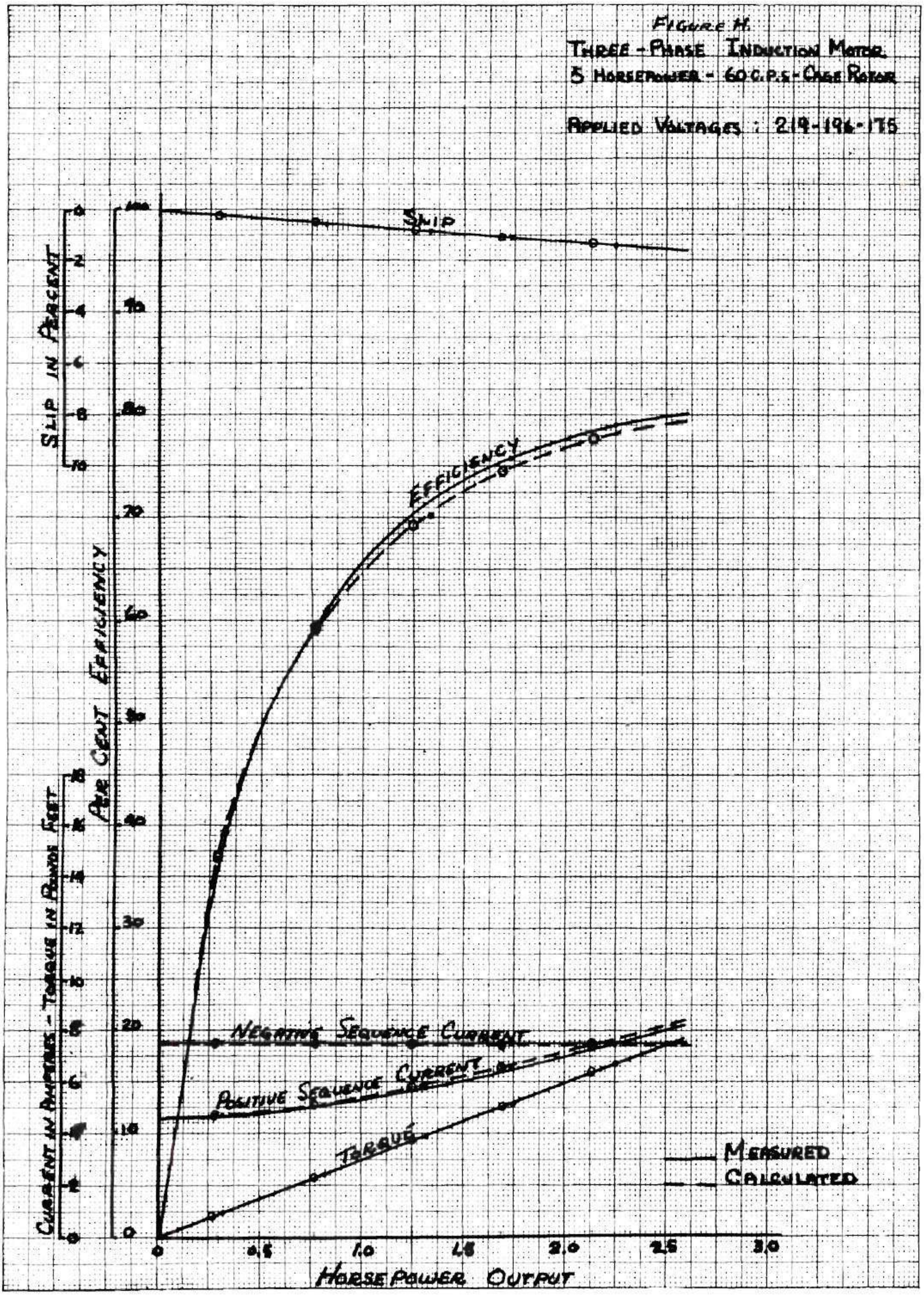


Figure H.
Three-Phase Induction Motor
5 Horsepower - 60 c.p.s. - Cage Rotor
Applied Voltages : 219-196-175
Slip
Efficiency
Negative Sequence Current
Positive Sequence Current
Torque
Measured
Calculated
Slip in Percent
Per Cent Efficiency
Current in Amperes - Torque in Pound Feet
Horsepower Output
0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18

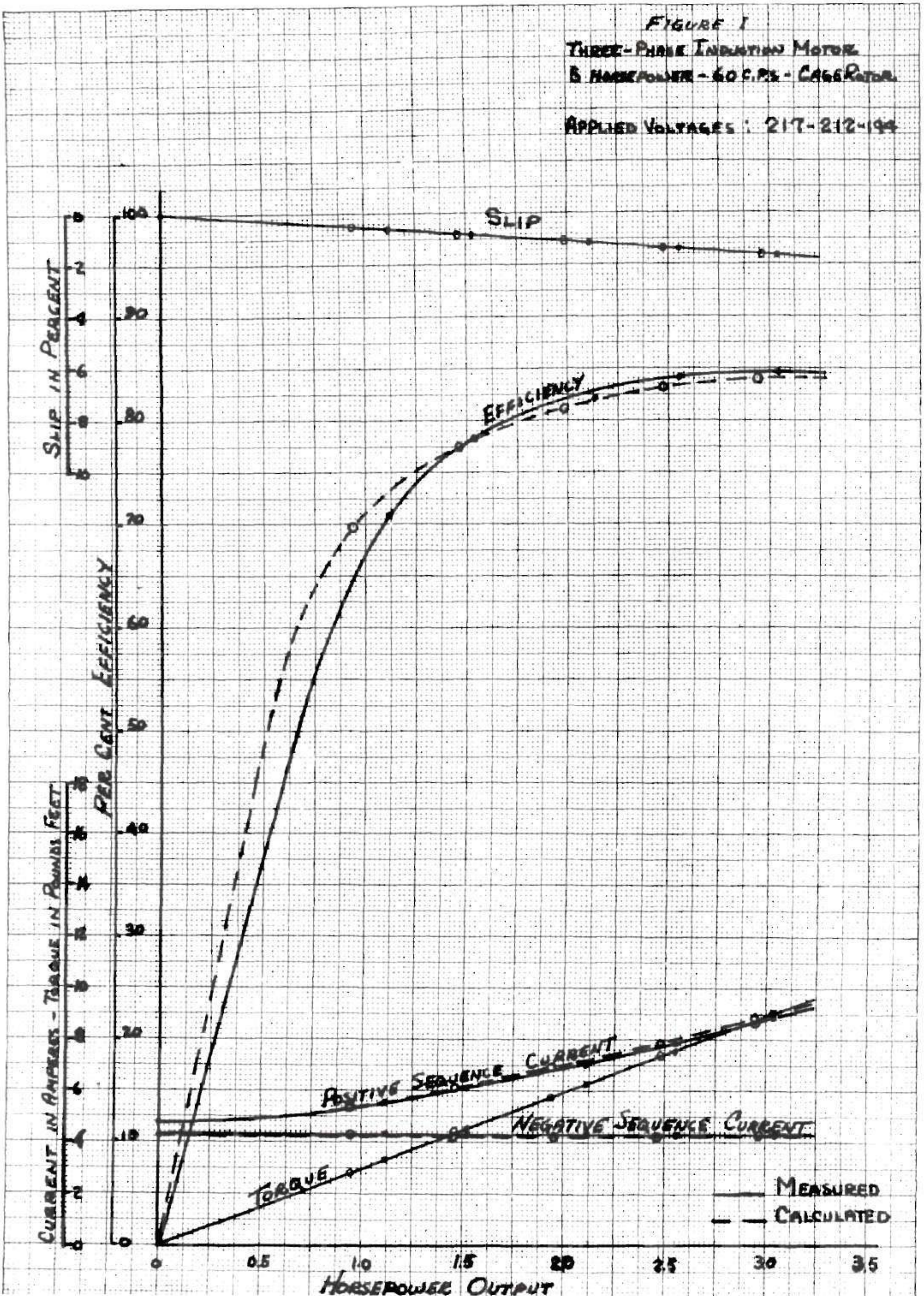


FIGURE I
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.P.S. - CAGE ROTOR
APPLIED VOLTAGES : 217-212-194
SLIP
EFFICIENCY
POSITIVE SEQUENCE CURRENT
NEGATIVE SEQUENCE CURRENT
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENT
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUNDS FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

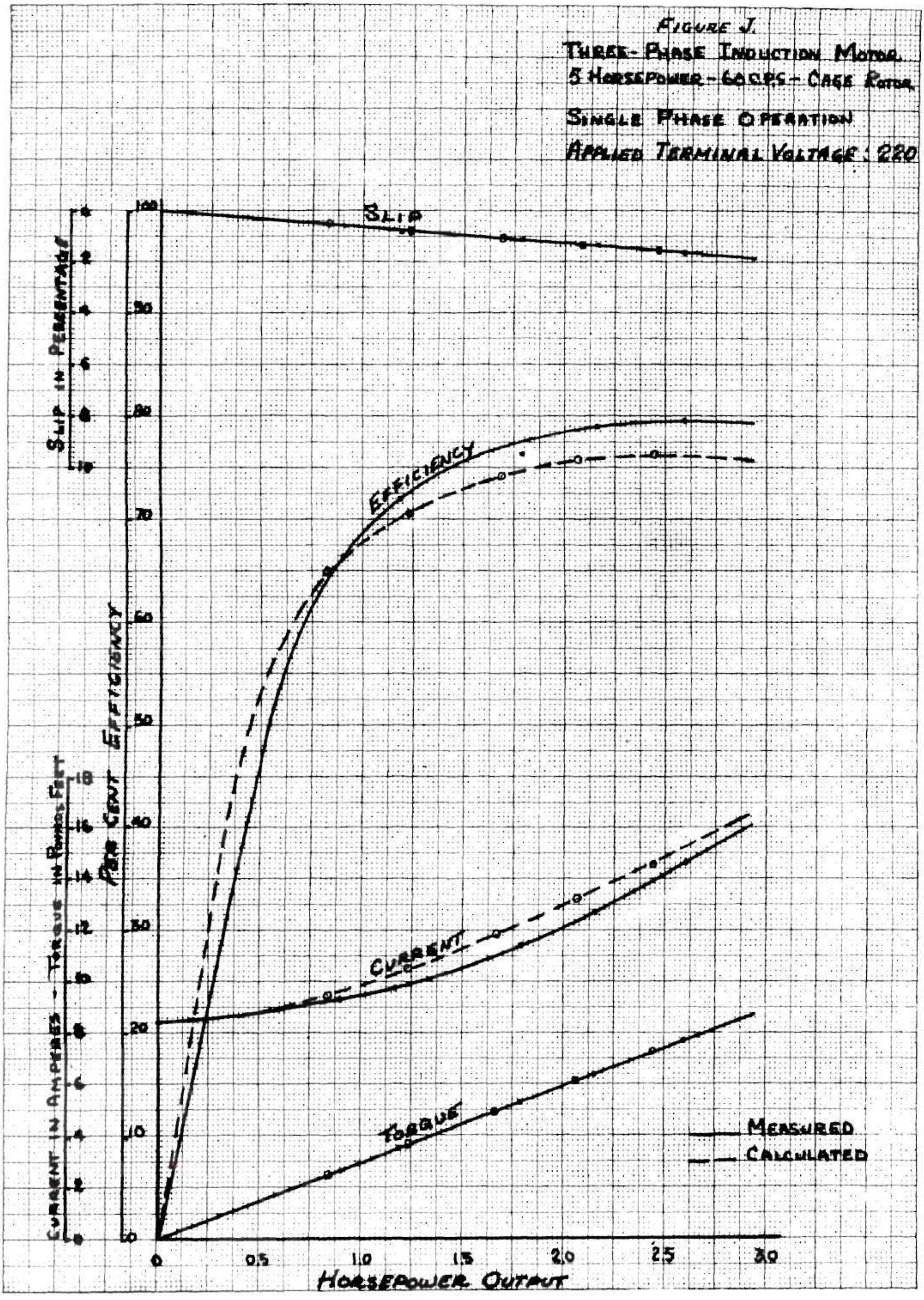


FIGURE J.
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER - 60 C.P.S. - CAGE ROTOR
SINGLE PHASE OPERATION
APPLIED TERMINAL VOLTAGE : 220
SLIP
EFFICIENCY
CURRENT
TORQUE
MEASURED
CALCULATED
SLIP IN PERCENTAGE
PER CENT EFFICIENCY
CURRENT IN AMPERES - TORQUE IN POUND FEET
HORSEPOWER OUTPUT
0
0.5
1.0
1.5
2.0
2.5
3.0

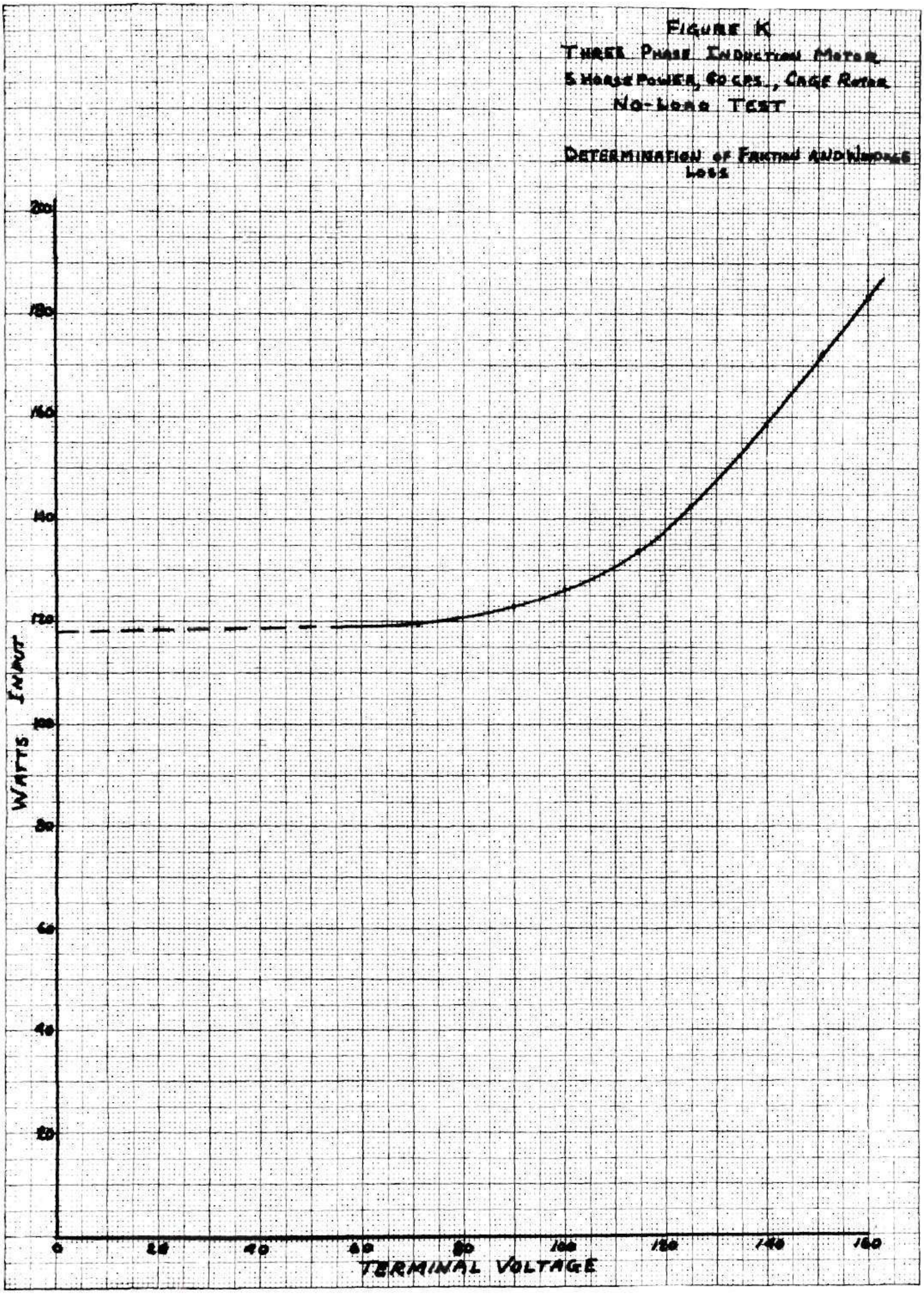


FIGURE K
THREE PHASE INDUCTION MOTOR
5 HORSE POWER, 60 CPS., CAGE ROTOR
NO-LOAD TEST
DETERMINATION OF FRICTION AND WINDAGE LOSS
200
180
160
140
120
100
80
60
40
20
WATTS INPUT
0
20
40
60
80
100
120
140
160
TERMINAL VOLTAGE

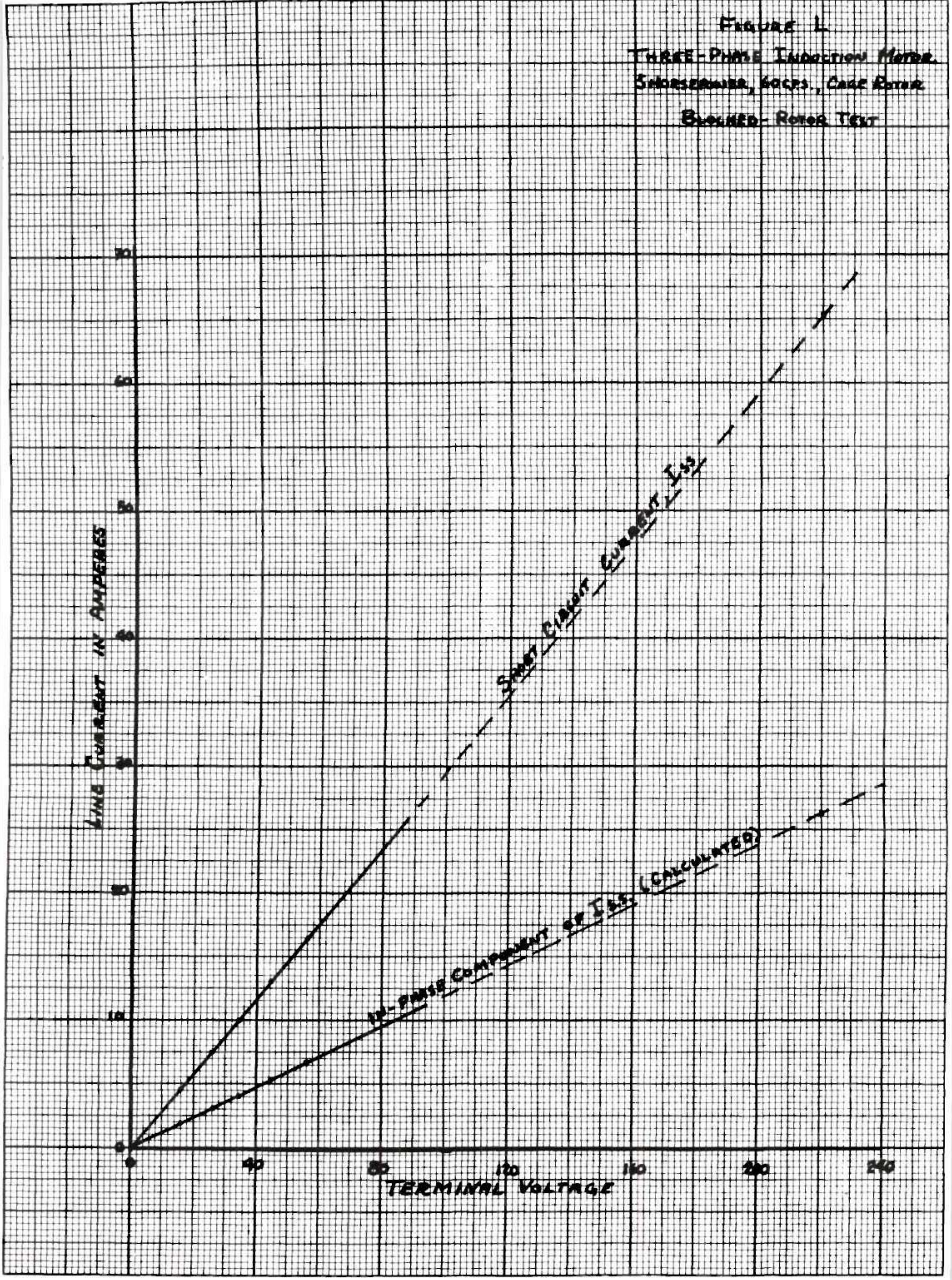


FIGURE L
THREE-PHASE INDUCTION MOTOR
5 HORSEPOWER, 60 CPS., CAGE ROTOR
BLOCKED-ROTOR TEST
SHORT CIRCUIT CURRENT I_{SS}
IN-PHASE COMPONENT OF I_{SS} (CALCULATED)
LINE CURRENT IN AMPERES
TERMINAL VOLTAGE
0
10
20
30
40
50
60
70
40
80
120
160
200
240

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